**Research article** **Научная статья**

https://doi.org/10.17073/2072-1633-2023-2-141-154

# Value equilibrium analysis based on the New Theory of Value. I

**Jie Wu**[1], **Zili Wu**[2] ✉
[1] *Institute of Shandong Development, Shandong University, 27 Shanda Road, Jinan 250100, China*
[2] *Guangzhou Milestone Software Co., Ltd.*
*(Economic Simulation Base of the National Simulation and Control Engineering Research Center),*
*Tower A-10B, Guangdong International Building, 339 Hunnish Dong Road, Guangzhou 510098, China*
✉ *wzl@gzmss.com*

**Abstract.** The (traditional) labor theory of value, founded by Smith with the basic axiom that labor determines value, and supplemented and improved by Marx with the axiom that labor creates surplus-value. Neoclassical economics newly studied the utility theory of value, but it abandoned the traditional labor theory of value and theory of surplus-value. Under the basic axiomatic system analogous to theoretical mechanics, the above three theories of value has been included and developed by the new theory of value, which established the value complex variable function including three basic quantities of quantity, quality and time, as well as labor-value, use-value and surplus-value. Starting from the value complex variable function, with the mathematical methods of Euler equation and Euler formula, this paper conduct a thorough and comprehensive analysis on the theory of value, under satisfactions of Marx's first and second laws, focusing on value equilibrium by giving the mathematical explicit expressions of both value equilibrium function and price equilibrium function in various forms applicable to the general commodity economic movement process, so as to prove the existence and stability of the extremum solutions to the corresponding functions.

**Keywords:** New Theory of Value, Marx's first law, Marx's second law, value equilibrium function, stability of extremum solution

**For citation:** Wu J., Wu Z. Value equilibrium analysis based on the New Theory of Value. I. *Russian Journal of Industrial Economics.* 2023;16(2):141–154. https://doi.org/10.17073/2072-1633-2023-2-141-154

# Анализ равновесия стоимости на основе Новой Теории Стоимости. I

**Цзе У**[1], **Цзыли У**[2] ✉
[1] *Институт развития Шаньдуна, Шаньдунский университет,*
*250100, Цзинань, Шанда роуд, д. 27, Китай*
[2] *Компания Guangzhou Milestone Software Co., Ltd. (База экономического моделирования Национального научно-исследовательского Центра моделирования и систем управления),*
*510098, Гуанчжоу, Ханниш Дон Роуд, д. 339, Башня А-10В, Китай*
✉ *wzl@gzmss.com*

**Аннотация.** Традиционная теория стоимости труда, разработанная Адамом Смитом и имеющая в своей основе аксиому о том, что труд определяет стоимость, была дополнена и усовершенствована Карлом Марксом с помощью аксиомы о том, что труд создает прибавочную стоимость. Неоклассическая экономика заново изучила теорию предельной полезности, но отвергала традиционную трудовую теорию стоимости и теорию прибавочной стоимости. В рамках базовой аксиоматической системы, аналогичной теоретической механике, три вышеупомянутые теории стоимости были объединены и получили дальнейшее развитие в новой теории стоимости, которая определяет, что функция комплексной переменной стоимости включает в себя три основные величины – количество, качество и время – так же, как и стоимость труда, потребительскую стоимость и прибавочную стоимость. Начиная с функции комплексной переменной стоимости, с помощью математических методов уравнения Эйлера и формулы Эйлера проведен тщательный и всесторонний анализ теории стоимости, при соблюдении первого и второго законов Маркса, сосредотачивая внимание на равновесии стоимости, давая математически точные выражения как функции равновесия стоимости, так и функции равновесия цены, в различных формах, применимых для общего процесса экономического движения товаров, чтобы доказать существование и устойчивость экстремальных решений соответствующих функций.

**Ключевые слова:** Новая Теория Стоимости, Первый закон Маркса, Второй закон Маркса, функция равновесия стоимости, устойчивость экстремальных решений

**Для цитирования:** Wu J., Wu Z. Value equilibrium analysis based on the New Theory of Value. I. *Экономика промышленности.* 2023;16(2):141–154. https://doi.org/10.17073/2072-1633-2023-2-141-154

## 基于新价值理论的价值均衡分析(I)

**吴 杰[1], 吴子立[2]** ✉

[1] 山东发展研究院, 山东大学, 山大路27号, 济南 250100, 中国

[2] 广州市长程软件有限公司 (中国国家仿真控制工程技术研究中心经济仿真基地), 广东国际大厦A附楼10B, 环市东路339 号, 广州 510098, 中国

✉ *wzl@gzmss.com*

**摘要:** 斯密根据劳动决定价值的基本公理假定创立了劳动价值理论。马克思进一步提出了劳动创造剩余价值的公理假定，补充和完善了劳动价值理论。新古典经济学创新地研究了效用价值理论，但是它却抛弃了传统的劳动价值理论和剩余价值理论。新价值理论在类比理论力学的基本公理假定体系下，将价值理论扩展到同时包含劳动价值理论、剩余价值理论和效用价值理论的范畴。同时，还建立了包含数量、质量和时间三个基本量，以及劳动价值、使用价值和剩余价值的价值复变函数。本文从价值复变函数出发，进一步采用成熟的欧拉方程和欧拉公式的数学方法对价值理论进行深入和全面的分析，在满足马克思第一定律和第二定律的条件下，分析两种基本的经济均衡——价值均衡和价格均衡——中的价值均衡，并给出适用于一般商品经济运动过程中各种不同形式的价值均衡函数的数学显式，以及证明相应函数的极值解的存在性和稳定性。

**关键词:** 新价值理论、马克思第一定律，马克思第二定律、价值均衡函数、极值解的存在性和稳定性

### 1. Introduction

There has been a long history of the study on the theory of value. According to Smith [1], labor determines value (hereafter Smith's law) Marx accepted Smith's law and added two basic axioms: first, the value of commodities is determined by the average labor necessary spent in the social production process under the balance of supply and demand (hereafter Marx's first law). According to Marx, "*The general value-form, which represents all products of labour as mere congelations of undifferentiated human labour.*" [2]. "*The total labour-power of society, which is embodied in the sum total of the values of all commodities produced by that society, counts here as one homogeneous mass of human labour-power, … so far as it has the character of the average labour-power of society, and takes effect as such; that is, so far as it requires for producing a commodity, no more time than is needed on an average, no more than is socially necessary. The labour-time socially necessary is that required to produce an article under the normal conditions of production, and with the average degree of skill and intensity prevalent at the time.*" [2]. In particular, the supply and demand will not affect the value of commodities, because "*In reality, supply and demand never coincide, or, if they do, it is by mere accident, hence scientifically = 0, and to be regarded as not having occurred. … In this way, the market-prices which have deviated from the market-values adjust themselves, as viewed from the standpoint of their average number, to equal the market-values, in that deviations from the latter cancel each other as plus and minus.*" [3]; second, "*the value created by using living labor*" is greater than "*the cost of labor power*" (hereafter Marx's second law)[1]. According to Jevons [4], Menger [5] and Walras [6], the value of commodities

[1] Marx [2] believed that in the production process, the force of labor would lead to value appreciation: "that part of capital, represented by labour-power, does, in the process of production, undergo an alteration of value. It … produces an excess, a surplus-value, which … I therefore call it the variable part of capital, or, shortly, variable capital".

is not determined by the amount of labor spent in the commodity production process, but by the utility of commodities in the consumption process for satisfaction of needs, and the utility of commodities is a function of diminishing marginal utility. It can be seen that in the history of economic theory, no matter how different understandings of commodity value by different economists, there is one common purpose to find a universal measure of value for different commodities [7].

In this regard, based on the traditional theory of value, as well as the hypothesis of Jevons, Tesla and Foley, right after its establishment, Newtonian mechanics has been introduced to explain the economic phenomena of human society. Early in the 17th century, Bacon, Hobbes, Locke and other mechanical materialists regarded the movement of all things in the world as or reduced to mechanical movement, which had gradually formed an entire economic theory since the middle of the 18th century, that is the "force of labor" – "force" in physics – determines the value of commodities and the "labor gravitational force" – "gravity" in physics – determines the surplus value of capital. Started from Smith, Marx, followed by Jevons [4] that the utility theory of value may be regarded as "*the mechanics of utility and self-interest*", and its research method is just as "*that of kinematics or statics*". Later, it was summarized by Tesla [8]: "*the general laws governing movement in the realm of mechanics are applicable to humanity*". More clearly, Foley [9] said: "*… the labor theory of value under the new interpretation plays a role in political economy analogous to the role played by Newton's laws in mechanics. The definition of the monetary expression of labor time is analogous to the stipulation in Newtonian mechanics that force is equal to mass multiplied by acceleration.*"

In recent years, some Chinese and Russian scholars have further adopted the mathematical paradigm of theoretical mechanics for reference to establish a mathematical model system for economics, which is called the New Theory of Value [10]. Compatible with the traditional theory of value, there are five[2] basic axioms:

*Axiom 1. The natural wealth obtained without labor has no value.*

*Axiom 2. The force of labor determines the value of commodities.*

*Axiom 3. The force of labor is equivalent with the reaction force of labor.*

*Axiom 4. There must be labor gravitational force in the labor process, which creates value appreciation.*

*Axiom 5. On the premise that the quality of commodities remains unchanged, there is an upper bound (saturation point or bliss point) for the demand amount for useful things.*

Starting from the above axiomatic system, the New Theory of Value further studies various important theoretical problems that have existed in the traditional theory of value for a long time, such as the universal measure of value under variable labor productivity, a comprehensive theoretical system of value including labor theory of value, theory of surplus-value and utility theory of value, and so on. With the above research, the new theory of value has making the following progresses:

– establishing a system of homogenous dimensions of value;

– finding the solution to value transformation with two compatible axioms of labor determining value and labor creating surplus-value;

– formulating a value complex variable function and the existence of its extremum solution;

– constructing the Euclidean metric space and Riemann metric space of commodity value, and exploring the universal measure of value under variable labor productivity and

– discussing the existence of equilibrium solutions to the commodity value function and market price function.

However, to be a complete scientific theoretical system, the new theory of value still has a long way to go with many basic theoretical problems that have not been studied. In this paper, we will focus on the existence and stability of extremum solutions to the value of commodities under homogeneous dimensions, which should consider both existence and stability of analytical solutions to these functions. Specifically, according to the new theory of value, the commodity value function can be expressed in the form of Euler equation, and the capital function in the form of Euler formula. Obviously, there are the analytical solutions to such functions. On this basis, we can further investigate the stability of extremum solutions to various functions.

There are two main methods for scientific research using mathematical theory: one is systematic and continuous; the other is local and discrete.

1. The former should highly abstract the problems studied, so that the theoretical problems of each branch can adopt different mathematical methods, and the research results expressed in different forms of mathematical theory can be

[2] There are 6 axioms in the New Theory of Value, where Axioms 2 and 4 can be converted equivalently under the principle of Dimensional Homogeneity, so that here they are combined into one.

converted to each other equivalently[3]. It is widely used in theoretical mechanics, with the advantage of generality in research results, and the disadvantage of great difficulty in abstract theoretical research.

2. The latter studies special and local problems, so that different problems studied separately as independent ones can be analyzed by different mathematical theories and methods. Instead, it does not require all research results to be converted to each other equivalently, that is, it does generally not require the general solution to the function.

Obviously, the latter one is relatively simple and easy, generally requiring more special necessary and sufficient conditions, and deducing less theoretical conclusions. Therefore, the latter method usually lacks of generality.

In this paper, we study the value equilibrium of commodities by analogy to the research methods of Euler equation and Euler formula used for Brachistochrone problem and similar theoretical problems in Newtonian mechanics, which is of great difficulty, but of generality in results, and helpful to establish a system of mathematical models based on the theory of value with homogenized labor value, use value and surplus value in a brand-new and higher level. It is worth noting that after the marginal revolution, neoclassical economics also adopted the mathematical method of Euler formula to study theoretical problems such as production function, where the research results of these related theories were suspended in the air, failed to landing on a solid foundation since neoclassical economics had abandoned the traditional labor theory of value and the theory of surplus value. In neoclassical economics, it was Debreu that clarified in-depth study on his celebrated "Theory of Value" [11], which analyzed the function of commodity value based on the theory of subjective utility value and individual preferences. It can prove the existence and stability of the extremum solutions to commodity utility value functions under partial order structures, however mathematically, such functions based on partial order can not obtain global maximum solutions. In this case, neoclassical economics cannot conduct an analysis on the value of commodities and capital with different material attributes under the principle of dimensional homogeneity. Therefore, when studying the value calculation of heterogeneous commodities, it is inevitable to encounter the Cambridge Controversies in Capital Theory of measuring and aggregating heterogeneous capital goods [12]. Additionally, in the modern choice theory, Arrow's impossibility theorem [13; 14] was given to prove that the measure of utility value in Arrow social welfare function [14] based on individual preference cannot be converted into that in Bergson-Samuelson social welfare function [15–17] based on utilitarianism in welfare economics [18]. Then, it was widely recognized that utility value cannot be measured [19]. Especially after this, modern economics evolved into a theoretical category with only the theory of price, but no theory of value.

Specifically, in neoclassical economics, the application of Euler formula started from Clark's research on the distribution of wealth since 1881 [20; 21], where the economic theoretical problem was whether the total revenue produced exactly equals the payment for all production factors if each unit of production factor receives compensation based on its marginal productivity. In 1894, Wicksteed [22] proposed that for production processes with constant returns to scale, the answer is YES. A few years later, Clark [23] gave the same conclusion as Wicksteed but more complete, which was the so-called "theory of marginal productive" or Euler theorem in neoclassical economics. In the following century, Euler theorem in neoclassical economics was widely applied, including Cobb-Douglas production function [24–26]; Tinbergen production function in econometrics [27]; The equilibrium model of the business cycle by rational expectation school [28]; Dynamic stochastic general equilibrium (DSGE) model in modern macroeconomics [29; 30], etc. However, above these are all based on the theory of price in neoclassical economics, which means that the return functions of product (or currency) quantity studied have overlooked the theory of value. Therefore, the mathematical form of Euler theorem in neoclassical economics is a first-order homogeneous function defined on convex function (or concave function satisfying the minimum of boundary conditions), without discussing the general solution and special solution of the general Euler equation – second-order homogeneous function, nor considering

[3] For example, the force defined by theoretical mechanics is a quadratic form function, which is converted into a second-order variable coefficient ordinary differential equation, or Euler equation for the study of continuous fluid mechanics problems. Then, Euler equation is converted into a variational equation to study the trajectory of a point, i.e. the geodesic – cycloid – in the Brachistochrone Problem. In addition, it can also be converted into a complex form, i.e. a natural logarithmic form of Euler's formula for the circular motion of particles. It can be seen that different mathematical methods adopted for specific problems actually are equivalent conversions from the same quadratic function of force, and there are also general solutions to differential equations. Therefore systematic and continuous methods have the advantages of generality and generality, but the necessary and sufficient conditions for solving general solutions to differential equations are relatively strict, with greater difficulty.

the value Euler formula based on the form of complex variable function.

In contrast, this paper examines the Euler equation based on constant coefficient in quadratic form and the Euler formula based on the complex variable function of value, focusing on the mathematical relationship among the quantity, quality, time, force of labor (labor power) [10], labor value, use value, value, constant capital, variable capital, surplus value, profit, rate of surplus value, profit rate and other factors of commodities. That is to say, in this paper, the value Euler equation in quadratic form defined in real domain has been mapped to a complex plane, so as to define the corresponding value Euler formula. Here, the domain of definition of exponential function of value Euler formula has been extended from real to complex. Therefore, the value of commodities has the mathematical forms of complex variable function, exponential function, logarithmic function and trigonometric function at the same time, so that it not only provides a comprehensive explanation for the theoretical problems related to commodity value, but also frees strict convexity constraints from the Euler formula by the new theory of value, making itself more general. In addition, its hyperbolic nature of complex variable function can provide a specific mathematical form of value base manifold for the differential fiber bundles based on the price – stock prices and futures indexes – of financial products constructed by Ilinski [31]. In this case, economics, based on the traditional theory of value, can further investigate prices, capitals, profits and stocks of commodities on the value tangent space attached to the value base manifold through continuous and differentiable value mapping. Finally, for the future research on the theory of value, modern differential geometry – related to Hamilton-Tian conjecture and Partial C0-conjecture [32] – can be introduced to study various complex economic theoretical problems, including the measure of value that are variable with time and inflation rate in the value complex manifold. In a word, with Euler equation in quadratic form and Euler formula based on complex variable function, the theory of value theory has established a rigorous and solid mathematical foundation, which enables it to introduce more cutting-edge mathematical tools for exploring its way forward in the future.

## 2. Analysis on value equilibrium of commodities

Concerning the value equilibrium of commodities, we use the research methods of Euler equation and Euler formula as follows: 1) to study a case typical and sufficient, with help to make the research results clear and easy for readers understanding the economic meanings of theoretical problems; 2) to obtain the general solution to the value Euler equation based on the dimensional analysis of heterogeneous commodities and capital goods under the principle of dimensional homogeneity; 3) to analyze the existence of the extremum solution to the value equilibrium function and the stability of the function when disturbed in the domain of definition.

### 2.1. Value Euler equation in simple circulation

When studying some physical problems, such as conduction of heat, vibration of circular membrane, propagation of electromagnetic waves, the equation

$$a\frac{d^2x}{dt^2}+b\frac{dx}{dt}+cx=f(x)$$

has been widely used, which is a second-order linear differential equation with variable coefficients, where the coefficient of the second derivative $dt^2$ is a quadratic function $ax^2$, the coefficient of the first derivative $dt$ is a quadratic function $bx$, and the coefficients $a$, $b$, $c$ of $x$ are constants. Such Euler equation can be transformed into homogeneous linear differential equation with constant coefficients through variable substitution, so the general solution to Euler equation can be obtained by solving characteristic equation, and then the characteristic solution by substituting the variables into the original Euler equation.

Then, we can use Euler equation to analyze the value function of a simple circulation of commodities as below:

$$m_b\frac{d^2b}{dt^2}+s_b\frac{db}{dt}+k_bb=f_b(t) \qquad (2.1.1a)$$

and

$$b_m\frac{d^2m}{dt^2}+s_m\frac{dm}{dt}+k_mm=f_m(t), \qquad (2.1.1b)$$

where $m$ is the quality of commodities, $b$ is the quantity of commodities, $m_b$, $b_m$, $e_b$, $e_m$, $k_b$ and $k_m$ are second-order, first-order and zero-order variable coefficients of Euler equation of commodity value respectively, $f_b(t)$ and $f_m(t)$ are the value Euler equations related to quantity and quality respectively.

Consider (2.1.1a) and (2.1.1b) together to investigate their general solutions, that is, (2.1.1a) and (2.1.1b) are the production and consumption processes of the same commodity, where the former is to create the labor value, and the latter is to realize the use value.

First, to solve the characteristic equation of (2.1.1a) by substituting variable $b = e^{mt}$, then $(b^2 + s_bb + k_b)e^{mt} = 0$. Since $e^{mt} \neq 0$, there is $b^2 + s_bb + k_b) = 0$, the characteristic root is

$$m_{1,2} = \frac{-s_b \sqrt{s_b^2 - 4k}}{2} = 0,$$

where are three solutions: 1) real root $m_1 \neq m_2$, with two linear independent characteristic solutions $b_1 = e^{m_1 t}$ and $b_2 = e^{m_2 t}$, and general solution $b = C_{11}e^{m_1 t} + C_{12}e^{m_2 t}$; 2) real root $m_1 = m_2$, with characteristic solution $b_1 = e^{m_1 t} = b_2 = e^{m_2 t}$, and general solution $b = (C_{11} + C_{12}t)e^{m_1 t}$; 3) a pair of conjugate complex roots $m_{1,2}$, $m_1 = \alpha_b + i\beta_b$ and $m_2 = \alpha_b - i\beta_b$, from Euler formula $e^{it} = (\cos t + i\sin t)$ and the theorem when $t = \pi$, $e^{i\pi} + 1 = 0$, then we have characteristic solutions $b_1 = e^{\alpha_b t}\cos\beta_b t$ and $b_2 = e^{\alpha_b t}\sin\beta_b t$, and general solution $b = e^{\alpha_b t}(C_{11}\cos\beta_b t + C_{12}\sin\beta_b t)$.

Second, to solve the characteristic equation of (2.1.1b) by substituting variable $m = e^{bt}$, then $(m^2 + s_m m + k_m)e^{bt} = 0$. Since $e^{bt} \neq 0$, there is $m^2 + s_m m + k_m = 0$, the characteristic root is

$$b_{1,2} = \frac{-s_m \sqrt{s_m^2 - 4k}}{2} = 0,$$

where are three solutions: 1) real root $b_1 \neq b_2$, with two linear independent characteristic solutions $m_1 = e^{b_1 t}$ and $m_2 = e^{b_2 t}$, and general solution $m = C_{21}e^{b_1 t} + C_{22}e^{b_2 t}$; 2) real root $b_1 = b_2$, with characteristic solution $m_1 = e^{b_1 t}$, and general solution $m = (C_{21} + C_{22}t)e^{b_1 t}$; 3) a pair of conjugate complex roots $b_{1,2}$, $b_1 = \alpha_m + i\beta_m$ and $b_2 = \alpha_m - i\beta_m$, from Euler formula $e^{it} = (\cos t + i\sin t)$ and the theorem when $t = \pi$, $e^{i\pi} + 1 = 0$, then we have characteristic solutions $m_1 = e^{\alpha_m t}\cos\beta_m t$ and $m_2 = e^{\alpha_m t}\sin\beta_m t$, and general solution $m = e^{\alpha_m t}(C_{21}\cos\beta_m t + C_{22}\sin\beta_m t)$.

**Definition 2.1.** For any commodity in a simple circulation, if the value per unit quality (2.1b) is equivalent to that per unit quantity, then the commodity has a value Euler equation with quality and quantity equivalence.

**Theorem 2.1.** If (2.1.1a) and (2.1.1b) of the same commodity satisfy $b = e^{mt}$ and $m = e^{bt}$ respectively, then (2.1.1a) and (2.1.1b) are the value Euler equations with equivalent quality and quantity[4].

**Proof:** From the general solutions to (2.1.1a) and (2.1.1b), if (2.1.1a) and (2.1.1b) are the same commodity and satisfy $b = e^{mt} \Rightarrow t = e^{mb}$ and $m = e^{bt} \Rightarrow t = e^{bm}$ respectively, then the variable substitutions to Euler equations of quality (2.1.1a) and quantity (2.1.1b) are equivalent, that is $b = e^{mt}$, $m = e^{bt}$, only when $b = m$. Also, the characteristic and general solutions to (2.1.1a) and (2.1.1b), as well as the constant terms, are all equivalent.

**Economic meaning:** This theorem shows that in a simple circulation, if the production and consumption process of the same commodity satisfy the requirements of Definition 2.1, then the commodity is a value Euler equation with equivalent quality and quantity[5]. In this case, the value of this commodity can be expressed through a unified value Euler equation, which generally indicates that, for the same commodity:

– in the production process, it takes the equivalent force of labor to produce one more unit of quantity as it does to improve one more unit of quality;

– in the consumption process, it gains the equivalent compensation for the force of labor by consuming one more unit of quantity or one higher unit of quality;

In other words, for any individual producer, the force of labor spent (labor value created) to produce one ton of white flour that meets the secondary national standard is equivalent to the force of labor compensated (use value realized) through consuming one ton of white flour that meets the secondary national standard.

## 2.2. Commodity value Euler equation in simple reproduction

Suppose that the production and consumption process of all commodities in a society is regarded as a process of production and consumption of total social products, if it is in a simple circulation with constant total value, then it is a simple circulation of economic movement. In particular, if it satisfies both

[4] In neoclassical economics based on the law of diminishing marginal utility, the economic Euler theorem refers to that if there are perfect competition in both product market and factor market, and constant returns of scale of producers, then under the condition of market equilibrium, the total amount of actual returns obtained by all production factors is exactly equal to the total social products. This is Clark's marginal productivity theory of distribution [23]. The economic Euler formula defined by Clark and the standard value formula defined in this paper both study the same economic problems, however based on different axioms and drawing different conclusions.

[5] In Capital, Marx divided labor into concrete labor (that creates use value) and abstract labor (that creates exchange value), and the exchange value of commodities depends on abstract labor, which is considered to be "the pivot on which a clear comprehension of Political Economy turns" [2]. Also, the measure of value of abstract labor is the "amount of labour-time ... on an average socially necessary" spent on products [2], which will be valid only when supply and demand are balanced, otherwise the market value will deviate from its value [3]. Boldly, with qualitative analysis, Marx concluded that the amount of labor-time is the universal measure of value of commodities. Here, we reproduce the process of labor time being a measure of value through rigorous mathematical derivation in Theorem 2.1, and prove that abstract labor time can indeed serve as the ultimate universal measure of value. However, such a measure of value is required to satisfy the strict conditions of Definition 2.1, otherwise there will be uncertainty for abstract labor time as the measure of value.

Marx's first and second laws, and generates both value and surplus-value, then it is a simple reproduction[6], which will be discussed in this section.

According to Theorem 2.1, the quality and quantity value Euler equations of a commodity are equivalent if they meet the conditions of Definition 2.1, let the equivalent commodity value Euler equation be the value Euler equation of a single quality-quantity dimension, denoted as

$$M_b \frac{d^2 B}{dt^2} + K_b \frac{dB}{dt} + L_b B^* = 0. \qquad (2.2.1)$$

Additionally, the value Euler equation of commodity includes two departments: means of subsistence (hereafter department I) and means of production (hereafter department II), denoted as:

$$\mathrm{I}: \widehat{M}_b \frac{d^2 \widehat{B}}{dt^2} + \widehat{K}_b \frac{d\widehat{B}}{dt} + \widehat{L}_b \widehat{B}^* = 0, \qquad (2.2.1a)$$

$$\mathrm{II}: \breve{M}_b \frac{d^2 \breve{B}}{dt^2} + \breve{K}_b \frac{d\breve{B}}{dt} + \breve{L}_b \breve{B}^* = 0, \qquad (2.2.1b)$$

where

$$\widehat{M}_b \frac{d^2 \widehat{B}}{dt^2}, \ \widehat{K}_b \frac{d\widehat{B}}{dt}$$

and $\widehat{L}_b \widehat{B}^*$ are the surplus-value, variable capital and constant capital of department I,

$$\breve{M}_b \frac{d^2 \breve{B}}{dt^2}, \ \breve{K}_b \frac{d\breve{B}}{dt}$$

and $\breve{L}_b \breve{B}^*$ are the surplus-value, variable capital and constant capital of department II.

**Theorem 2.2.** (1) (2.2.1a) and (2.2.1b) should satisfy:

$$\widehat{M}_b \frac{d^2 \widehat{B}}{dt^2} + \widehat{K}_b \frac{d\widehat{B}}{dt} = \breve{L}_b \breve{B}^*,$$

or

$$\breve{M}_b \frac{d^2 \breve{B}}{dt^2} + \breve{K}_b \frac{d\breve{B}}{dt} = \widehat{L}_b \widehat{B}^*.$$

(2) (2.2.1a) and (2.2.1b) constitute a simple reproduction process.

**Proof:** 1. According to Marx's definition of simple reproduction, it is easy to prove that (2.2.1a) and (2.2.1b) satisfy

$$\widehat{M}_b \frac{d^2 \widehat{B}}{dt^2} + \widehat{K}_b \frac{d\widehat{B}}{dt} + \widehat{L}_b \widehat{B}^* = \breve{L}_b \breve{B}^* + \breve{L}_b \breve{B}^*$$

and

$$\breve{M}_b \frac{d^2 \breve{B}}{dt^2} + \breve{K}_b \frac{d\breve{B}}{dt} + \breve{L}_b \breve{B}^* =$$

$$= \widehat{M}_b \frac{d^2 \widehat{B}}{dt^2} + \widehat{K}_b \frac{d\widehat{B}}{dt} + \breve{M}_b \frac{d^2 \breve{B}}{dt^2} + \breve{K}_b \frac{d\breve{B}}{dt}.$$

2. From Theorem 2.1 that (2.1.1a) and (2.1.1b) are the value Euler equations with equivalent quality and quantity, that is, the labor value produced in any economic circulation is equivalent to the use value consumed. Therefore, this repeated economic circulation is also a process of conservation of value.

**Economic meaning**: In the simple reproduction composed of production and consumption process, the value created in the production process of department I is labor value, and that created in the consumption process of department II is use value. Therefore, during the economic circulation, labor value and use value are the production and consumption repeated and developed in the movement of conservation of value.

### 2.3. Euler formula for capital motion in simple reproduction

If the value of the total social product in a simple reproduction process satisfies Definition (2.1), which is an Euler equation of equivalent commodity value, and its basic quantities – time, quantity, and quality – are all one unit quantity with homogeneous value dimension[7], then we will take the value movement process of this commodity as a standard Euler formula for capital motion (hereafter standard capital formula. Here, we use a method similar to Sraffa's definition for the standard measure of value, and prove the existence of the universal measure of value – "standard commodity". However, this "standard commodity" still needs to satisfy many strict mathematical conditions. That is to say, in order to solve the difficult problem of the universal measure of value under variable labor productivity proposed by Ricardo, in his book "Commodity Production by Means of Commodities" [33], Sraffa constructed a special standard commodity, which satisfies a certain rate of surplus-value and has a fixed and unchanging physical form in the process of capital motion, including labor commodity, means of subsistence commodity, and means of production commodity, and regards this "standard commodity" as the universal measure of value, unique, general and

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[6] According to Marx [3], the simple reproduction is an economic circulation of conservation of value, where the circulation of capital not only brings surplus value, but also maintains the conservation of value. Therefore, let c be constant capital, v be variable capital, m be surplus-value, department I be industries for production of means of production, and department II be industries for production of articles of consumption, the formula for conservation of value in a simple reproduction among the social reproductions is: $\mathrm{I}(c + v + m) = \mathrm{I}(c) + \mathrm{II}(c) \Rightarrow \mathrm{I}(v + m) = \mathrm{II}(c)$.

[7] In other words, the quantity and quality of products have the homogeneous value dimension analogous to the property of "isotropy" in physics.

constant. Here, the research thinking and direction of Ricardo and Sraffa's attempt to find the universal measure of value are both correct, yet neither of them introduced Riemannian geometry. Mathematically, there exists a universal measure of value – the Riemann metric – under variable labor productivity [34]. For Ricardo, unfortunately his life ended before the establishment of Riemannian geometry in 1854. However, when Sraffa studied the measure of value, he still turned to a simple and rough way by constructing "standard commodity" in physical form, which is not that satisfactory.

Therefore, we need to examine the existence and stability of the extremum points of the capital Euler formula.

Analogous to physics, a standard capital formula can be seen as the disk motion formed by the trajectory of a ball rotating around its center. To be specific, assuming that in a special environment where the mass of any object remains constant, there is a ball with a mass of $m$ rotating on the disk, connected by a rope of a length $r$ with one end fixed at the center of the disk (also the center of rotation). Assuming there is no friction, the rotation of the ball is due to the pull of the rope. To the observer who rotates with the disk, the ball is stationary subjected to both the tensile force of the rope and the centrifugal force, which are equivalent in magnitude and opposite in direction, and the resultant force is zero according to Newton's law. Therefore, the value Euler equation of single quality-quantity dimension that satisfies (2.2.1) can be regarded as an equation that satisfies Newton's second law:

$$f = ma = m\omega^2 r = mv^2/r = m4\pi^2 r/T^2, \quad (2.3.1)$$

where $f$ is the centrifugal force, $a$ is the centripetal acceleration, i.e. $a = \omega^2/r = 4\pi^2 r/T^2$, $m$ is the constant of commodity quality, $\omega$ is the angular velocity of the circular motion, $r$ is the radius of the circular motion, $T$ is the period of the circular motion, and $\pi$ is the ratio of circumference. Here, the radius of the circular motion of commodity value in a standard capital formula is 1, indicating the period (a cycle), mass (quality), quantity, and value are all one unit in commodity value motion.

Note that if you calculate the logarithm of a complex number, i.e. the logarithm of $z = r(\cos\theta + \sin\theta)$, it is specified that $\theta$ is the principal argument angle of $z$. Then, let the value complex number $w = b + im$, which maps $w$ to $z$ through an exponential function $z = e^w$, that is, the $w$-th power of $z = e$. Since $r(\cos\theta + \sin\theta) = e^b(\cos\theta + \sin\theta)$, from the definition of complex equality, $r = e^b$, $\theta = 2k\pi$, $k \in Z$, there is $b = \ln r$, $m = \theta + 2k\pi$, i.e. $w = \ln r + i(\theta + ik\pi) = \ln r + i\theta + 2k\pi i$. From this, take $w = \ln z$ as a logarithmic function, which is multi-valued in complex numbers, with one independent variable versus multiple dependent variables, and countless branches. Specifically, when $k = 0$, $w = \ln r + i\theta$ is referred to as the principal value branch of the logarithmic function, written as $w = \ln z$, that is, the real part of value $w$ is the natural logarithm of $z$, and the imaginary part is the principal argument angle $\theta$ of $z$, where $r \neq 0$.

To sum up, we can use a complex plane to describe the uniform circular motion of commodity value with a radius of $r$ and an angular velocity of $\omega$, where the velocity is $t$. Let the independent variable be acceleration $a$, and the dependent variable be angular velocity $\omega$. The commodity value function is $f(a) = re^{\omega(t)}$, where

$$\omega = \frac{d\theta}{dt},$$

velocity $v = r\omega$, and angular acceleration

$$a = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}.$$

Therefore, $f'(a) = i\omega re^{i\omega(t)}$ is the linear velocity, which is the position vector from the origin to $t$, and the imaginary number $i$ is rotated 90 degrees counterclockwise. $f''(a) = -\omega^2 re^{i\omega(t)}$ is the angular acceleration. Then, substituting (2–6) obtains that $f''(a) = -\omega^2 re^{iw(t)} = -\omega^2 rm = -\omega^2 r4\pi^2/T^2$, where $e^{iw(t)} = m = 4\pi^2/T^2$. Therefore, the centrifugal force plus the centripetal force equals zero, that is, $f = ma = m\omega^2 r = mv^2/r = m4\pi^2/T^2$.

According to the new theory of value, the value of commodities is a complex variable function, with the real part being the labor value function and the imaginary part being the use value function, both of which are determined by the acceleration of the quantity or quality – force of labor in the production or consumption process. Therefore, assuming that the simple economic circulation is a simple reproduction that satisfies (2.3.1) with the variable capital $V = 1$ and the ratio of variable capital and constant capital as constantly $1:3.14 = \pi$, where the constant capital is $C = 3.14$, total advanced capital is $\varpi = 1 + 3.14 = 4.14$, the rate of surplus-value of each capital circulation is 100%, i.e. $\kappa = M/C = 100\%$. That is to say, the surplus-value $M = 1$ and the average rate of profit $\varpi = M/(C + V) = 1/4.14 = 24\%$ will be obtained in one circulation of capital per unit time. Then, with unlimited times of capital circulations repeated per unit time using the same advanced capital, the average rate of profit ultimately obtained is a stable constant. In this case, it is appropriate for us to take Euler formula as standard capital formula, mathematically, a complex variable function in the form of $e^{ix} = \cos x + i\sin x$, where $e$ is the base of natural logarithm, $i$ is the imaginary unit, and $x$ is arbitrary real number, as the independent variable of natural logarithm.

Therefore, we can transform the standard capital formula into a complex variable function of commodity value $e^{if} = \cos f + i\sin f$. Specifically, if there is a value mapping

$$w = \frac{1}{f}$$

for each time $t$ in interval $a \le t \le b$, it can be seen as a composite mapping of labor value and use value:

$$f = \frac{1}{f_1},\ \ w = \overline{f_1},$$

where $f$ is the labor value, and $\overline{f}$ is the use value. Let $f$, $f_1$, $w$ be points on the same complex plane, then $w = \overline{f_1}$ is a symmetric mapping about the real axis, that is,

$$f_1 = \frac{1}{\overline{f}}$$

maps $f$ to $f_1$, and its argument is the same as $f$:

$$Arg\, f_1 = -\mathrm{Arg}\,\overline{f} = Arg\, f \qquad (2.3.2)$$

and the module

$$|f_1| = \left|\frac{1}{f_1}\right| = \frac{1}{|\overline{f_1}|} = \frac{1}{|f_1|}$$

satisfies $|f_1||f| = 1$. Let a circle with a center as the origin and a radius of 1 be the unit circle, then (2.3.2) is called the symmetric mapping about the unit circle, and $f$ and $f_1$ are the mutual symmetric points about the unit circle, shown as below:

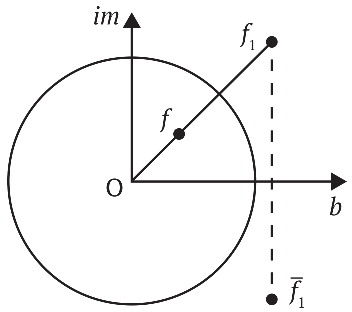


**Fig. 1**

Where $b$ is the quantity and a real number, $m$ is the quality, and $i$ is the imaginary number[8]. Further, value mapping

$$w = \frac{1}{f}$$

maps any point outside the origin to another point, that is

$$\begin{cases} w = w_+ + iw_- \\ w_+ = u(f_{1b} + f_{1m});\ \ iw_- = iv(f_{2b} + f_{2m}); \\ f_{1b} = m_{1b}\dfrac{d^2b_1}{dt^2};\quad f_{1m} = b_{1m}\dfrac{d^2m_1}{dt^2}; \\ if_{2b} = im_{2b}\dfrac{d^2b_2}{dt^2};\ \ if_{2m} = ib_{2m}\dfrac{d^2m_2}{dt^2}, \end{cases} \qquad (2.3.3)$$

where $u(f_{1b} + f_{1m})$ and $iv(f_{2b} + f_{2m})$ are respectively the real and imaginary parts of the value composite mapping, $f_{1b}$ and $f_{1m}$ correspond to $f_1$, $f_{2b}$ and $f_{2m}$ correspond to $\overline{f}$, respectively in Fig. 1.

Note that according to the research method of statics, if force of labor is viewed as a uniform acceleration process, the displacement distance of commodity particles in the commodity vector space represented by the dynamic equations $u(f_{1b} + f_{1m})$ and $iv(f_{2b} + f_{2m})$ of value – changes in the quantity or quality level of the product – can be expressed by the parallelogram rule. In other words, the amount of force of labor is equal to the modulus of commodity value [35], i.e., the labor value of quantity and quality of commodities is expressed as

$$f_{1b} = \sqrt{a_b^2 + b_b^2} \ \ \text{and}\ \ f_{1m} = \sqrt{a_m^2 + b_m^2},$$

and the use value as

$$f_{2b} = \sqrt{a_b^2 + b_b^2} \ \ \text{and}\ \ f_{2m} = \sqrt{a_m^2 + b_m^2}$$

respectively.

Obviously, in polar coordinates, the complex variable functions of labor value and use value in (2.3.3) in the form of statics can be expressed respectively as $(r_1, \theta_1)$ and $(r_2, \theta_2)$ by module length and argument. For the complex variable functions of labor value $u(b_1, im_1)$, there are

$$|r_1| = \sqrt{b_1^2 + m_1^2} \ \ \text{and}\ \ \theta_1 = \arctan\left(\frac{b_1}{m_1}\right).$$

And for the complex variable functions of use value $v(m_2, ib_2)$, there are

$$|r_2| = \sqrt{b_2^2 + m_2^2} \ \ \text{and}\ \ \theta_2 = \arctan\left(\frac{b_2}{m_2}\right).$$

Here, the multiplication of complex variable functions is expressed as the addition of arguments and the multiplication of module lengths, and the logarithmic operations of $(r_1, \theta_1)$ and $(r_2, \theta_2)$ are expressed as $\ln(r_1, \theta_1) = \ln(r_1, i\theta_1)$ and $\ln(r_2, \theta_2) = \ln(r_2, i\theta_2)$. Therefore, the multiplication of complex variable functions of labor value and use

[8] Here, according to Theorem 2.1, the quality and quantity of products have attained the universal measure of value under the principle of dimensional homogeneity. Therefore, the quantitative coefficients $f_{1b}$, $f_{2b}$ and qualitative coefficients $f_{1m}$, $f_{2m}$ in (2.3.3) are assumed to be constants by people's daily experience in specific situations.

value indicates that the value complex variable function runs one circulation, as well as the magnitude of the rate of changes in value – surplus-value or profit – and the corresponding amount of surplus-value or profit amount. Then, the process of capital circulation can be expressed as Euler formula:

$$e^{i\theta} = \cos\theta + i\sin\theta,$$

where

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e = 2.718281828459\dots.$$

That is to say, the base $e$ of natural logarithm is given by an important limit. $e$ is an infinite non recurring decimal number and a transcendental number. Therefore, we can define when $n$ approaches infinity, it is assumed that the value of commodities created in a simple reproduction is a standard unit of value, and in each economic circulation, there is surplus-value generated by the acting force of improving skill and dexterity of workmen – the labor gravitational force. Therefore, it is worthy to discuss whether there is a maximum value for the increase of wealth value with the increasing times of circulations of standard capital per unit time, and whether the maximum value is stable.

**Lemma.** If a complex variable function $f$ is holomorphic and harmonic, then $f$ satisfies the following theorems:

– *Pontryagin's Maximum Principle: If K is a compact subset of U, then the function induced by f on K can only reach its maximum and minimum values on the boundary. If U is connected, then this theorem means that f cannot reach its maximum or minimum unless it is a constant function.*

– *Intermediate Value Theorem: Let B(b,m) be a ball completely in U with b as the center and r as the radius, then the average value of the harmonic function f(b) obtained on the boundary of the ball is equivalent with the average value of f(b) obtained inside the ball.*

– *Liouville Theorem: If f has defined harmonic functions in the whole real domain R with a maximum or minimum value, then f is a constant function.*

**Theorem 2.3. (**1) If the value complex variable function $f(w) = u(b, m) + v(b, m)i$ of standard capital expressed as (2.3.3) satisfies the conditions of Cauchy-Riemann Equations and is harmonic, then it must have an extremum solution, that is, (2.3.3) satisfies the simple reproduction condition of (2.3.1); (2) The maximum rate of surplus-value in (2.3.3) is stable that, within any unit time, ensuring that each circulation of a simple reproduction can obtain the equivalent rate of surplus-value, and, with unlimited times of capital circulations repeated, the final maximum value of standard capital obtained is a constant in the form of natural logarithm, i.e.

$$e^{b+im} = \lim_{f\to\infty} r(\cos\theta + i\sin\theta) \approx w. \qquad (2.3.4)$$

**Proof: (**1) Existence

First, the Cauchy-Riemann equations of $u(b, m)$ on a pair of real valued functions $u(b, m)$ and $v(b, m)$ include:

$$\frac{\partial u}{\partial b} = \frac{\partial v}{\partial m} \text{ and } \frac{\partial u}{\partial m} = -\frac{\partial v}{\partial b},$$

which is a necessary condition for a function to be differentiable at a point.

Since the value function $w = w(f) = u(b, m) + v(b, m)i$ is defined in region $D$, the necessary and sufficient conditions for its resolution in $D$ are:

– $u(b, m)$ and $v(b, m)i$ are differentiable everywhere within $D$;

– $u(b, m)$ and $v(b, m)i$ satisfy first-order partial differential equations

$$\frac{\partial u}{\partial b} = \frac{\partial v}{\partial m} \text{ and } \frac{\partial u}{\partial m} = -\frac{\partial v}{\partial b},$$

everywhere within $D$.

This is a first-order homogeneous equation, which indicates that in a simple reproduction, the labor productivity in the production process to increase quantity and that to improve quality are mutually offset.

Second, the condition for harmonic equation

$$\frac{\partial^2 f}{\partial b^2} + \frac{\partial^2 f}{\partial m^2} = 0$$

is a second-order homogeneous equation, which indicates that in a simple reproduction, the sum of the value generated by the acceleration of increasing quantity and that of improving quality is equal to zero, and the result of interaction between labor value and use value created in a circulation of simple reproduction is equal to zero.

Therefore, if the value complex variable function is Cauchy-Riemann equations, i.e. holomorphic, also harmonic, then according to the above Lemma, there must be the extremum solution to the value complex variable function. This means that the value complex variable function must satisfy the conditions of simple reproduction, that is, the labor value of product produced in the production process is just equivalent with the use value of product consumed in the consumption process.

(2) Stability

Based on the discussions in (2.3.1), (2.3.3) and (2.3.4) above, it is assumed that the value circulation affected by external factors is a standard capital formula, with the total advanced capital consisting of constant capital and variable capital. Also, let variable capital be 1, constant capital be 3.14, total advanced capital be 4.14, and the rate of surplus-value generated by capital motion be 100 %, the value circulation can be expressed as a value Euler formula:

$$w = e^{rif} = r(\cos f + i \sin f),$$

where the force of labor $f$ is an independent variable, the value $w$ is a dependent variable, and the variable capital $r$ is a constant coefficient.

Therefore, the inverse function of natural logarithms satisfies a very important property:

$$g(f) = \frac{f}{dw} = \frac{1}{\frac{df}{dw}} = \frac{1}{(\ln f)'} = f,$$

$$f' = g'(f) = \frac{df}{dw} = f, \quad g(0) = 1,$$

that is, the derivative of this function is itself, and when $f = 0$, $w = 1$, written as $\exp(f)$. Therefore, $\exp(f) = e^f$, $(e^f)' = \exp(f)$.

At the same time, during the motion of force of labor, due to the labor gravitational force, the variable capital with a rate of surplus-value of 100% will generate a factorial that:

$$\exp(f) = 1 + \frac{f}{1!} + \frac{f^2}{2!} + \frac{f^3}{3!} + \cdots$$

where one circulation of capital is one order, then the repeated circulations of capital generate surplus-values in a compound interest calculation method: before a circulation, variable capital is principal, i.e. 0! = 1; then it's snowballing interests on each circulation, so 1! = 2, 2! = 4, 3! = 8..., so that we can use Euler formula with the argument θ as the independent variable to express the standard capital formula. Here, from Kurtz formula $i\theta = \ln(\cos(\theta) + i\sin(\theta))$, in particular, if $\theta = \pi$, there is the Euler identity:

$$e^{i\pi} = \cos\pi + i\sin\pi = -1,$$

$$e^{i\pi} + 1 = 0.$$

Here, the most important constants in mathematics are linked together: two transcendental numbers: the base $e$ of natural logarithm and the pi; two units: the imaginary number unit $i^2 = -1$ and the natural number unit 1, as well as the common mathematical unit 0. When $\theta = \pi$, $e^{i\theta} = -1$ means the use value of one unit, $e^{\theta} = 1$ means the labor value of one unit, and $e^{i\theta} + 1 = 0$ means that the use value of commodities has been realized in the consumption process, or the labor value of force of labor has been expended in the production process.

Therefore, when $r = 1$ means the force of labor of constant capital in a simple reproduction process, we can define the capital Euler formula as:

$$e^w = r(\cos\theta + i\sin\theta).$$

To be specific, from (2.3.3), there is a complex variable function among the value and force of labor of commodities, and the quantity and quality of products. If $w = b + im$, then there is

$$\lim_{f\to\infty}\left(1+\frac{w}{f}\right)^f = \lim_{f\to\infty}\left(\left(1+\frac{w}{f}\right)^{\frac{f}{w}}\right)^w =$$

$$= \left(\lim_{w\to\infty}\left(1+\frac{w}{f}\right)^{\frac{f}{w}}\right)^w = e^w.$$

Also, from

$$\lim_{f\to\infty}\left(1+\frac{w}{f}\right)^f = e^w \Rightarrow e^{b+im} =$$

$$= \lim_{f\to\infty}\left(1+\frac{f+ib}{f}\right)^f \Rightarrow e^{b+im} = \lim_{f\to\infty}\left(\frac{f+ib}{f}+i\frac{m}{f}\right)^f,$$

let

$$\left(\frac{f+b}{f}+i\frac{b}{f}\right)^f = r(\cos\theta+\sin\theta),$$

according to the definitions of modulus and radian, and the de Moivre's formula, it can be obtained that

$$r = \left[\left(\frac{f+b}{f}\right)^2 + \left(\frac{m}{f}\right)^2\right]^{\frac{f}{2}},$$

$$\theta = f\arctan\left(\frac{m}{f+b}\right).$$

Further, from

$$\lim_{f\to\infty}\ln r = \lim_{f\to\infty}\frac{f}{2}\ln\left(1+\frac{2b}{f}+\frac{b^2+m^2}{f^2}\right) =$$

$$= \lim_{f\to\infty}\frac{f}{2}\left(\frac{2b}{f}+\frac{b^2+m^2}{f^2}\right) = b \Rightarrow \lim_{f\to\infty} r = e^b,$$

and

$$\lim_{f\to\infty}\theta = \lim_{f\to\infty} f\arctan\left(\frac{m}{f+b}\right) =$$

$$= \lim_{f\to\infty}\frac{mf}{f+b} = \lim_{f\to\infty} m - \frac{bm}{f+b} = m,$$

there is

$$e^{b+im} = \lim_{f\to\infty}\left(\frac{f+b}{f}+i\frac{m}{f}\right)^f = \\ = \lim_{f\to\infty} r(\cos\theta+\sin\theta) = e^b(\cos m + i\sin m). \quad (2.3.5)$$

If $m = 0$, that is, all commodities express their value only by quantity, and the quality of commodities is expressed as a constant coefficient, then there is

$$e^{bi} = \cos m + i\sin m,$$

$$e^{\theta i} = \cos\theta + i\sin\theta.$$

From the definition of complex equality, there is

$$e^{fi} = \cos f - i\sin f,$$

$$e^{-fi} = \cos f + i\sin f.$$

If $f = \pi$, there is Euler identity

$$e^{\pi i} = 1,$$

$$e^{\pi i} - 1 = 0.$$

Therefore, with the cyclical changes of $f$ or $\theta$, the Euler formula for capital value is a mathematical graph that rotates around the origin with the definition domain reciprocating in [–1,1]. This property of the generalized capital Euler formula is very appropriate to illustrate the nature of the standard capital formula.

In summary, the simple reproduction process of commodities is a circular motion, the production price function of capital satisfies the basic relationship of trigonometric functions, which can be solved by Euler's formula. Let variable capital $V = f$, then force of labor $f$ as modulus $r$ equals to 1, that is, $f = r = 1$. From the definition of standard capital, constant capital is 3.14, total advanced capital is 4.14, the rate of surplus-value of variable capital be 100%, and the amount of surplus-value is 1, then defined by simple reproduction above, if the capital circulation repeats unlimited times per unit time, then from (2.3.5), the extremum solution of the surplus-value is convergent, i.e. when $m = 0$, it is

$$e^{i\theta} = (\cos\theta + i\sin\theta),$$

where the times of capital circulations are integer greater than 1, variable capital is 1, and the rate of surplus-value is 100%, then the total return on capital is

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e = 2.718281828459\ldots,$$

given the average rate of profit of capital = total surplus-value / total social capital, denoted as $\varpi = \delta/M$, i.e.

$$\varpi = \frac{\delta}{M} \approx \frac{(e-1)}{M}.$$

If there are unlimited circulations of capital invested within a unit time of one year, and the rate of surplus-value of each circulation is 100%, the maximum of the average rate of profit of standard capital is:

$$\varpi = \frac{\delta}{M} \approx \frac{(2.71-1)}{4.14\cdot\%} \approx 41\%,$$

which is a stable constant.

**Economic meaning:** It is known that the value of commodities created by capital in a simple reproduction process is a constant, and in each economic circulation there is surplus-value generated under the labor gravitational force. Then, with unlimited times of economic circulations of standard capital repeated per unit time, according to Euler formula, there must be a stable maximum value for the value appreciation of wealth. To put it plainly, mankind are intelligent to improve the skill and dexterity themselves, so that in every repeated simple reproduction, rational people will correct wrong behaviors to stabilize the movement of economic system. Therefore, it must be a stable system as long as the social production is a repeating process of simple reproductions that remain unchanged for a long time. In particular, the maximum of this stable system satisfies the conclusion of Euler formula that it is a natural number in the form of an irrational number.

## 3. Conclusion

In summary, through the study on the existence and stability of extremum solutions for the value function of commodities that satisfy supply and demand balance by analogy to the research methods of Euler equation and Euler formula in Newtonian mechanics, this paper has established not only a system of mathematical models that maintains logical consistency with Marx's simple reproduction and Sraffa's standard commodity, but also a theoretical foundation for mathematical forms based on the theory of commodity price. After marginal revolution, economics has only the theory of price, instead of the theory of value. This abnormal situation will be completely changed in the future. Specifically, the abstract economic laws in value calculation revealed through rigorous mathematical reasoning in this paper are not a game of pure theoretical deduction. On the contrary, these abstract theoretical conclusions will have extraordinary significance in guiding the socio-economic practices. For example, since the Industrial Revolution in the 18th century, overcapacity and periodic economic crises have been persistent nightmares for human society, which have not been dealt with effectively by both classical economics and neoclassical economics. The new theory of value, with a theorem that labor productivity has inverse ratio to product quality [35; 36], proposed that producing more high-quality products during overcapacity will help to eliminate excess capacity and re-balance the supply and demand of economic system at a higher quality level, which has provided a scientific theoretical basis for the Chinese government to implement high-quality macroeconomic management over the past decade, and has achieved significant economic benefits. Obviously, if the strategy of high-quality development can be widely applied worldwide, modern human society will greatly reduce international trade frictions caused by trade deficits and financial debt crises caused by overcapacity. In other words, the fundamental reason for the two world wars in the 20th century and the worldwide trade wars, science and technology wars, financial wars, mixed wars and Russia-Ukraine conflict in recent years is rooted in different countries scrambling for the world market to consume excess capacity. If guided by a scientific theory of value and there is no overcapacity and periodic economic crises, then the root cause of vicious competition for the world market and even the risk of the doom of men by nuclear war will be eliminated. Compared with any significant research achievements in natural sciences, it is not to be outdone by the contribution and significance of abstract research on the theory of value in economics for reducing costs of trials and errors and creating value of wealth in human social practices.

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## Information about authors

**Jie Wu** – Researcher, Institute of Shandong Development, Shandong University, 27 Shanda Road, Jinan 250100, China; e-mail: jw@gzmss.com

**Zili Wu** – Engineer, Guangzhou Milestone Software Co., Ltd. (Economic Simulation Base of the National Simulation and Control Engineering Research Center), 10B, Tower A, Guangdong International Building, 339 Huanshi Dong Road, Yuexiu District, Guangzhou 510098, China; e-mail: wzl@gzmss.com

Поступила в редакцию **16.02.2023**; поступила после доработки **29.05.2023**; принята к публикации **06.06.2023**

Received **16.02.2023**; Revised **29.05.2023**; Accepted **06.06.2023**