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Price equilibrium analysis based on the new theory of value*

Jie Wu[1,2], Zili Wu[2] ✉

[1] *Institute of Shandong Development, Shandong University, 27 Shanda Road, Jinan 250100, China*

[2] *Guangzhou Milestone Software Co., Ltd. (Economic Simulation Base of the National Simulation and Control Engineering Research Center), Tower A-10B, Guangdong International Building, 339 Hunnish Dong Road, Guangzhou 510098, China*

✉ *wzl@gzmss.com*

Abstract. The universal measure of value has been always controversial in the traditional theory of value. Analogous to the research method of theoretical mechanics, developed from the traditional theory of value – labor theory of value, theory of surplus value and utility theory of value, the new theory of value has established a value complex variable function with labor value and use value as unknown functions, further in the mathematical logic of Euler equation and Euler formula, provided a universal measure of commodity value. On this basis, this paper focuses on the mathematical explicit expressions of value, exchange value, price and market equilibrium function that satisfy the principle of dimensional homogeneity, and the proof of the existence and stability of extremum solutions of market price equilibrium function, so as to help economics become a scientific theoretical system, integral with consistent logic, formed by qualitative theories, mathematical models, and computer models.

Keywords: new theory of value, dimensional homogeneity, value, exchange value, price, market price equilibrium function, complex systems, artificial intelligence

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Анализ равновесной цены, основанный на новой теории стоимости

Цзе У[1,2], Цзыли У[2] ✉

[1] *Институт развития Шаньдуна, Шаньдунский университет, 250100, Цзинань, Шанда роуд, д. 27, Китай*

[2] *Компания Guangzhou Milestone Software Co., Ltd. (База экономического моделирования Национального научно-исследовательского Центра моделирования и систем управления), 510098, Гуанчжоу, Ханниш Дон Роуд, д. 339, Башня А-10B, Китай*

✉ *wzl@gzmss.com*

Аннотация. В традиционной теории стоимости всегда вызывало споры понятие универсальной меры стоимости. Авторы по аналогии с методами исследования, нашедшими применение в теоретической механике, в добавление к традиционной теории стоимости, использующей такие положения, как трудовая теория стоимости, теория прибавочной стоимости и теория стоимости полезности, ввели понятие новой теории стоимости. Взята система основных аксиом и математических моделей, причем функция зависимости стоимости включает три основные независимые переменные – количество, качество и время, – и параметры – трудовая стоимость, потребительская стоимость и прибавочная

* Price equilibrium analysis carried out in the article is based on the concept of the New Theory of Value and is a continuation and development of the research, contained in the article Jie Wu, Zili Wu Value equilibrium analysis based on the New Theory of Value. I, where the main provisions of the concept were formulated and applied (Russian Journal of Industrial Economics. 2023;16(2):141–154. https://doi.org/10.17073/2072-1633-2023-2-141-154).

стоимость. В работе использованы математические расчеты с применением уравнений Эйлера и формулы Эйлера и получена универсальная мера стоимости товара. При этом исследование сфокусировано на равновесии цен. В нем приведены математические выражения функции равновесия рыночных цен, удовлетворяющие принципу размерной однородности и доказывающие существование устойчивости ее экстремальных значений.

Ключевые слова: новая теория стоимости, размерная однородность, стоимость, обменная стоимость, цена, функция равновесия рыночной цены, сложные системы, искусственный интеллект

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基于新价值理论的价格均衡分析

吴 杰[1,2]，吴子立[2] ✉
[1] 山东发展研究院, 山东大学, 山大路27号, 济南 250100, 中国
[2] 广州市长程软件有限公司 (中国国家仿真控制工程技术研究中心经济仿真基地),
广东国际大厦A附楼10B, 环市东路339 号, 广州 510098, 中国
✉ *wzl@gzmss.com*

摘要：在传统的价值理论中，统一价值尺度的问题一直是一个存在争议的问题。新价值理论类比理论力学的研究方法，将传统的价值理论扩展到同时包含劳动价值理论、剩余价值理论和效用价值理论的范畴，建立了一个以劳动价值函数和使用价值函数为未知函数的价值复变函数，并进一步采用欧拉方程和欧拉公式的数学方法统一了衡量商品的价值尺度。本文在以上研究成果的基础上进一步研究满足量纲一致性原理的价值、交换价值、价格和市场均衡函数的数学显式，以及证明市场价格均衡函数的极值解的存在性和稳定性。显然，这些理论问题的解决，有助于经济学成为一个科学的理论体系，使得定性理论、数学模型和计算机模型形成一个具有逻辑一致的整体。

关键词：新价值理论，量纲一致性，价值、交换价值、价格、市场均衡函数，复杂系统人工智能

1. Introduction

Mature economics must be a systems engineering, consisting of conceptual models, mathematical models, and computer models [1]. The conceptual model solves the theoretical problems of qualitative analysis, the mathematical model solves the problems of theoretical logic inference and proof using the mathematical language with uniqueness, and the computer model solves the problems of transforming the theory into technology to replace human physical and mental labor, including economic decision-making in complex systems. In the new theory of value [2], we study the measure of value and homogenized dimension of commodities, initially to ensure that the value and price calculations of the mathematical model are reasonable and correct, and ultimately is to establish an effective computer model. Without a scientific theory of value, there cannot be a reasonable computer model in economics, which cannot become an AI product applicable in complex systems.

For a long time, a universal measure of value remains unsolved in economics, leading to a fundamental drawback of the computer models established by various economic schools that cannot perform accurate value calculations and cannot be converted into AI products to assist human in decision-making in complex systems. Specifically, current popular economic computer models have the following shortcomings:

(1) It is impossible to analyze various heterogeneous capital goods under the principle of dimensional homogeneity. For example, the production functions in Econometric Model (EM)/Computable General Equilibrium (CGE)/Dynamic Stochastic General Equilibrium (DSGE)/Agent-based Modelling (ABM) models have only two types of independent variables, capital and labor, which cannot distinguish the difference in value dimensions among different production factors with different material attributes, and cannot analyze the problem of heterogeneous capital [3] under the principle of dimensional homogeneity.

(2) It is unable to establish an effective closed-loop control system when using complex theory to

study a unified economic system, resulting in the divergence of simulation experiments after multiple iterations in the vast majority of ABM models with heterogeneous agents, so that it is impossible to simulate the balanced operating state of the economic system [4]. Specifically, ABMs for Simulation of social economy were established from bottom-up, attempting to simulate the integrated macro and micro economies. The early representative ABM was an agent-based simulation model of the U.S. economy (ASPEN) developed by Sandia National Laboratories in the 1990s. It was a computer system of rule-based model from bottom-up. Although this model abandoned the rigorous assumptions of neoclassical economics on the subject behavior and interaction, it did not take scientific economic theory as the modeling mechanism, so it failed to build a model that can simulate the general law of the integrated macro-micro economic movement. Especially, due to the lack of a regulation mechanism for global equilibrium, it always ran to be halted by unexplainable divergence, and simulation results only reflected tendency and could not mirror the reality [5]. Earlier in this century, it was disappointing for DSGE's failure in prediction, especially in predicting financial crises [6], so that many scholars turned to ABM for the development of new macroeconomic models. According to Wang et al. [3], the current medium-sized ABM macroeconomic models can be divided into seven categories: (a) AGH model [7]; (b) CATS model (Complex Adaptive Trivial Systems model) [8]; (c) EUBI model (Eurace@Unibi model) derived from EURACE [9; 10]; (d) EUGE model, which is the frame model of EURACE [11]; (e) JAMEL model (Java Agent-based Macro-Economic Laboratory model) [12]; (f) KS model ("Keynes Meeting Schumpeter" frame) [13]; (g) LAGOM model [14]. Bounded rationality and endogenous market disequilibrium rather than perfect rationality and market equilibrium assumed in mainstream macroeconomics are introduced to establish highly coupling agent-based models with integrated macro-micro economies. These models generally adopt heuristics or rules of thumb based on qualitative optimization of behavior, or based on experimental and empirical evidence [15]. These models are currently widely applied to analysis of fiscal, monetary, macro prudential regulation, labor market, and climate policies [6].

(3) It fails to exchange data under the principle of dimensional homogeneity between different types of models established by different economic schools, and to integrate these models with each other when studying a universal economic system by classification method [4]. For example, various mainstream macroeconomic theoretical models, such as the large-scale EM/CGE/DSGE models, have achieved many theoretical and applied results. These models formulate simultaneous equations started from the traditional Keynesian macroeconomic theory, the neoclassical general equilibrium theory, and the neoclassical and neo-Keynesian macroeconomic theories, ultimately will be applied to economic structure analysis, economic forecasting, and policy evaluation after the estimation and calibration in model parameters [6]. However, the traditional Keynesian theory lacks micro-agents, and the micro-foundation of the neoclassical and neo-Keynesian theories are usually built by homogenous agents. Consequently, EM/CGE/DSGE models fail to directly describe the behaviors of heterogeneous micro agents, so that their simulation results can not match well with the corresponding micro scenarios. In addition, in CGE/DSGE models, usually the micro-agents are assumed to have perfect rationality or rational expectation, that cannot reflect the bounded rationality or adaptive behaviors of real world agents [16]. Finally, CGE is an equilibrium model, ignoring the process of dynamic adjustment in disequilibrium. Although DSGE is an integrated model of equilibrium and disequilibrium, the process of disequilibrium is usually caused by exogenous disturbances, which ignores the inherent instability of the economic system, e.g. the nonlinear interaction between economic agents [17].

In summary, due to the above drawbacks, these popular economic computer models are unable to convert into AI products that assist human in economic decision-making in complex systems. However, these drawbacks have been remedied by an ABM model based on the new theory of value – systems model for simulation of Social Economy Dynamics (SED model) [4]. Taking the new theory of value as the modeling basis analogous to Newtonian mechanics, SED is developed to be a super large scale agent-based model for global economic simulation. It is the first ABM model that has solved value calculation, so as to not only simulate various economic events in the integrated macro and micro economies, but also generate digital twin systems that highly mirror the real systems of international, national and regional economies. The current SED model allows the Internet access with a user-friendly interface (https://www.gzmss.com) and the high performance parallel computing with China's Tianhe-2 supercomputer. In recent years, the SED model has been applied to simulate the impact of International incidents such as the Sino-US trade war on the global economy, and has been recognized gradually [18–21]. Later, the SED model will be integrated with ChatGPT based on natural language, becoming a new generation of complex system AI model based on mathematical language, capable of assisting human in economic decision-making.

In the future, the SED model will be developed into a large AI model for complex systems, where the key lies in its adoption of the new theory of value as the modeling mechanism. Specifically, based on the traditional theory of value – labor theory of value, theory of surplus-value and utility theory of value, as well as the hypothesis of Jevons, Tesla and Foley, in recent years, some Chinese and Russian scholars have further adopted the mathematical paradigm of theoretical mechanics for reference to establish a mathematical model system for economics, which is called the new theory of value [2]. Further, Wu and his team explore the universal measure of value more deeply and systematically, starting from Marx's abstract labor time and Sraffa's standard commodity, through the mathematical methods – Euler equation and Euler formula – to define the measure of commodity value in both dynamic and static forms. The former is for the value of a commodity expressed through the Euler equation, while the latter is a standard capital formula for the aggregate social products (simple reproduction) expressed through the Euler equation [22]. The above theoretical research results have proven the rationality of both the abstract labor time in Marxist economics [23; 24] and the standard commodity system in post Keynesian economics [25] as the measures of value, which endow with a rigorous mathematical form to the universal measure of value, promoting the development and improvement of modern economics.

It can be seen that on the basis of above achievements, we can further study more important theoretical problems about value. In this paper, we will focus on market price equilibrium by giving the mathematical explicit expressions of price equilibrium function under satisfactions of Marx's first and second laws, and proving the existence and stability of its extremum solutions, based on the strict mathematical definitions and expressions with homogeneous dimensions provided for economic concepts including the value, unit value, exchange value, and price of commodities, and the analysis on how the market supply and demand maintain a balance by a mandatory approach of profit maximization under the spontaneous market adjustment mechanism that the market price of commodities deviates from the value. Obviously, if we can comprehensively and deeply understand the above theoretical conclusions, it will be beneficial for us to create economic AI models.

2. Basic economic concepts with dimensional homogeneity

If each commodity is given certain dimension of value under the principle of dimensional homogeneity and exchanged equivalently in the actual commodity market, then the value and price of each commodity can be accurately measured under complex conditions. In this regard, we will specifically analyze various economic concepts with this dimensional homogeneity.

2.1. Basic properties of commodity value dimension

It is known that the value of commodities is a complex variable function based on second-order homogeneous equation, if it is also holomorphic and harmonic, then it satisfies the maximum theorem, intermediate value theorem and Liouville theorem [22]. On this basis, we can further discuss the theoretical problem related to dimensional homogeneity of commodity value, that is, how to exchange commodities of different physical properties equivalently in the market under the principle of dimensional homogeneity. Obviously, it first needs to determine the property that the dimension of each commodity can be expressed as constants.

Liouville Lemma (Integral function/entire function) refers to a function that is analytical everywhere on the entire complex plane. If the entire function $f(z)$ is bounded on the entire plane, that is, if it satisfies the inequality $|f'(z)| \leqslant M$ for all z, then $f(z)$ must be a constant.

Corollary 2.1. If the complex variable function of commodity value is an entire function $f(z) = u(b, m) + iv(b, m)$ and there is a real number M, $v > M$, such that $z \in C$, then $f(z)$ is a constant.

Proof: $\because f(z)$ is an entire function,

$\therefore if(z) = iu(b, m) - v(b, m)$ is also entire. If $F(z) = e^{if(z)}$, then $F(z)$ is also entire.

And $\because |F(z)| = e^{-v} < e^{-u}$, from Liouville theorem, $F(z)$ is a constant.

$\therefore f(z)$ is also a constant.

Economic meaning: If the value of commodities is a constant, that is, the circulation motion of capital is a simple reproduction process, then the commodity value function will be a constant. In other words, in a stable socio-economic system based on simple reproductions, the commodity value function can be abstracted as a value complex variable function, of which the extremum solution satisfies Liouville lemma. Therefore, common commodities will be endowed with certain units of measurement established due to economic systems or accepted due to customs, such as cloth in feet or meters, grain in kilograms, and gold in grams.

2.2. Value of a commodity with dimensional homogeneity

According to the new theory of value [2], the value of commodities is determined by the force of labor, which is a vector, so that the forces of labor spent for producing values of different commodities cannot be compared with each other in amount. Therefore, it is necessary to convert the force of labor into the amount of value in the form of energy. For example, commodities can be compared in amount by value, that is by comparing the displacements of the quantity and quality of products in the commodity vector space during the production process, i.e., the vector modulus of the force of labor [26], or the work done by the force of labor, i.e., the kinetic energies of value [2]. Obviously, for any commodities, only the measure of value is transformed into a scalar from the vector of the force of labor can it be endowed with dimensional homogeneity. To simplify the discussion below, we assume that the step for universal measure of value has been done, that is, with dimensional homogeneity the amount of value embodied in each commodity has been known. Then, the problem left to be discussed is only how to compare and commensurate the values of different commodities by the universal measure of value.

To be specific, if the value of commodities is a complex variable function based on Euler's equation, also satisfies Liouville theorem, then its solutions, including general and characteristic, will be the solutions to a constant coefficient second-order ordinary differential equation. In this case, all commodities should follow the principle of dimensional homogeneity, that is, the quality and quantity standards of all commodities will have homogeneous dimensions, which ensures equivalent exchanges among commodities.

Definition 2.2.1. For a commodity, let t be the time related to the commodity production, m be the quality standard, k be the unit of measurement, l be the consumer population, and f_b be the value function for the force of labor related to quantity (hereafter the quantity-value function). When the quality, unit of measurement and the consumer population for the commodity are constants, the labor value function can be expressed as

$$w_b = f_b(b) = m_b \frac{d^2b}{dt^2} + k_b \frac{db}{dt} + l_b b. \qquad (2.2.1a)$$

If the quantity of the commodity is constant while the quality is variable, then the labor value function will be expressed as

$$w_m = f_m(m) = b_m \frac{d\ m}{dt} + k_m \frac{dm}{dt} + l_m m, \qquad (2.2.1b)$$

where b is the quantity standard, k_m is the unit of measurement for commodity quality, l is the consumer population, and f_m is the value function for the force of labor related to quality (hereafter the quality-value function).

Then, according to the new theory of value, the value function of any commodity is a second-order homogeneous linear equation. [22] Also, such Euler equation mathematically can be transformed into homogeneous linear differential equation with constant coefficients through variable substitution, so the general solution can be obtained by solving characteristic equation, and then the characteristic solution by substituting the variables into the original Euler equation. For example, in the value Euler equation with quality as a constant, there are three solutions:

(1) real root $m_1 \neq m_2$, with two linear independent characteristic solutions $b_1 = e^{m_1 t}$ and $b_2 = e^{m_2 t}$, and general solution $b = C_{11}e^{m_1 t} + C_{12}e^{m_2 t}$;

(2) real root $m_1 = m_2$, with characteristic solution $b_1 = e^{m_1 t} = b_2 = e^{m_2 t}$, and general solution $b = (C_{11} + C_{12}t)e^{m_1 t}$;

(3) a pair of conjugate complex roots $m_{1,2}$, $m_1 = \alpha_b + i\beta_b$ and $m_2 = \alpha_b - i\beta_b$, from Euler formula $e^{it} = (\cos t + i\sin t)$ and the theorem when $t = \pi$, $e^{i\pi} + 1 = 0$, then we have the characteristic solutions $b_1 = e^{\alpha_b t}\cos\beta_b t$ and $b_2 = e^{\alpha_b t}\sin\beta_b t$, and the general solution $b = e^{\alpha_b t}(C_{11}\cos\beta_b t + C_{12}\sin\beta_b t)$.

Therefore, in the value Euler equation that either quality or quantity is a constant, we can use the general solution to determine the unified dimension, or unified value metric, of a commodity in a simple circular process. Here, the so-called simple cycle refers to the realization of the value of the same commodity in the cycle process of production and consumption, including the labor value and use value of the commodity.

To be specific, if there are n commodities, $\alpha = 1, 2, \ldots, n$, where the homogeneous dimension of value of any α^{th} commodity can be converted from (2.2.1a) and (2.2.1b) into the relation of supply-demand balance as below[1]:

$$\begin{aligned} & m_b \frac{d^2b}{dt^2} + k_b \frac{db}{dt} + l_b b^* = 0 \\ & \Rightarrow m_b \frac{d^2b}{dt^2} + k_b \frac{db}{dt} = l_b b^* \\ & \Rightarrow m_b k_b \left[(1 + df_b) \frac{db}{dt} \right] = l_b b^*, \end{aligned} \qquad (2.2.2)$$

[1] According to (2.1.1a), (2.1.1b) and (2.2.1) in Value Equilibrium Analysis based on the New Theory of Value by Wu & Wu [22].

which is the differential equation of value of the α^{th} commodity with a constant quality that achieves a supply-demand balance, that is, (2.2.2) indicates the value differential equation that satisfies the constraint that the total quantity per unit time is equal to the amount of rational demand for the commodity[2]. Here, the coefficients in (2.2.2) show value dimensions of certain quantitative units based on standard quality, where m_b is the qualitative dimension of commodity value, namely the amount of value of one commodity – rice – with quality level of A; k_b is the quantitative dimension, namely one kilogram rice; and l_b is the dimension of market demand for the commodity, namely the amount of rice that satisfies 10,0000 consumers for one year.

At the same time, let the product quality in the process of social economic movement be $m = m^*$, which means the commodity quality is equal to the amount of rational demand[3], the labor value function of the Euler value differential equation can be expressed as the imaginary part of the value complex variable function, that is

$$ib_m \frac{d^2m}{dt^2} + k_m \frac{dm}{dt} + l_m m = 0$$
$$\Rightarrow ib_m \frac{d^2m}{dt^2} + k_m \frac{dm}{dt} = -l_m m^* \qquad (2.2.3)$$
$$\Rightarrow ib_m k_m \left[(1 + df_m) \frac{dm}{dt} \right] = -l_m m^*.$$

which is the differential equation of value of the α^{th} commodity with a constant quality that achieves a supply-demand balance, that is, (2.2.3) indicates the value differential equation that satisfies the constraint that the total quality per unit time is equal to the amount of rational demand for the commodity. Here, the coefficients in (2.2.3) show value dimensions of certain qualitative units based on standard quantity, where b_m is the quantitative dimension of commodity value, namely the amount of unit value of one commodity – gasoline – with quantitative standard of 10,000 barrels; k_m is the qualitative dimension, namely 95#gasoline; and l_b is the dimension of market demand for the commodity, namely the amount of gasoline that satisfies 10,0000 consumers for one year.

Obviously, the value of any commodity is uncertain until it has the dimension In other words, with dimensions, the value of commodities can be uniquely determined, that is, the quantity, quality and value of products that satisfy the supply-demand balance in any economic system from the base year and every subsequent year can be all calculated. For example, in an economic system, when it is a process of simple reproduction, the sum of the labor value of social products created during production and the use value realized during consumption is equal to zero. Mathematically from Euler equation, there are always the characteristic and general solutions to the value Euler equation of any commodity. Therefore, in simple circulation of economic movements, the homogeneous dimension of commodity value can always be determined appropriately due to the stability of economy.

Definition 2.2.2. Let there be n commodities, $w_\alpha = (w_{1\alpha}, w_{2\alpha}) \in W_\alpha^2$ be the vector of force of labor for the α^{th} commodity, $\alpha = 1, 2, \ldots, n$, where $w_{1\alpha}$ is the value related to quantity, $w_{2\alpha}$ is the value related to quality. Then, from (2.2.1a) and (2.2.1b), in the vector space of value of the α^{th} commodity, with the amount of value of $w_\alpha = (w_{1\alpha}, w_{2\alpha}) \in W_\alpha^2 \cong R^2$, then the relationship between the value and the acceleration of quantity and quality of the commodity will be

$$w_{1\alpha} = f_{1\alpha}(b) = m_b \frac{d^2b}{dt^2}$$

and (2.2.4)

$$w_{2\alpha} = f_{2\alpha}(m) = b_m \frac{d^2m}{dt^2},$$

where w is the scalar of value, f_1 is the force of labor function related to quantity, f_2 is the force of labor function related to quality, m_b is the quantitative coefficient (scalar) of the quality acceleration, and b_m is the qualitative coefficient (scalar) of the quantity acceleration[4].

To be specific, given the dimension of force of labor of commodities is [MLT^{-2}] [2], let there be n commodities, $f_\alpha = (f_{1\alpha}, f_{2\alpha}) \in F_\alpha^2$ be the commodities

[2] The balance between supply and demand is a necessary condition for determining the value of any commodity, according to Marx's first law.

[3] This is a specific manifestation of human demand, that is, assuming that the quantity of products is constant, what quality of products do people need to satisfy their needs. The specific manifestation of this demand is effective when there is a continuous conversion function between product quality and quantity. For example, a man needs to drive for a distance of 1000 kilometers, given the volume of gasoline, there is a certain quality standard for the purity of gasoline – the percentage of octane content.

[4] Note that for the same commodity, $m_b = b_m$ means the quantity b in the production process equal to the quantitative coefficient mb of the quality acceleration; meanwhile, $b_m = m_b$ means the quality m in the production process equal to the quantitative acceleration b_m [2]. This requires strict conditions, mainly that a dimensional system of commodity value that meets the principle of dimensional homogeneity in commodity activities has been gradually established.

of subscripted divisions 1 and 2 (vectors of quantity and quality) belonging to the α^{th} commodity of n dimensions, $\alpha = 1, 2, \ldots, n$. Then, let $b_\alpha = x_{1\alpha}$ and $m_\alpha = x_{2\alpha}$, for the α^{th} commodity, the units of measurement for both quantity and quality are dimensions of energy yet dimensionless. However, for the same commodity, the units of measurement for unit quantity and unit quality are different. For example, rice can be represented by quantity in volume and by quality in weight, according to the principle of dimensional homogeneity for commodity value, we need to provide a conversion coefficient for the equivalent value of quantity in volume and quality in weight of rice. That is to say, it needs to homogenize the dimensional units of force of labor per unit quantity and that per unit quality of the same commodity. In the case of dimensionless, the conversion coefficient will be

$$[\mathrm{MLT}^{-2}]_{x_{1\alpha}}[\mathrm{MLT}^{-2}]^{-1}_{x_{2\alpha}} = \nu_{(x_{1\alpha}, x_{2\alpha})},$$

which means that for the same commodity, the dimensional units of value per unit quantity and that per unit quality are homogenized[5]. Then, let $\nu_{(x_{1\alpha}, x_{2\alpha})} = \nu$, the sum of forces of labor related to quantity and quality of any α^{th} commodity can be expressed as

$$f_\alpha = f_{1\alpha} + f_{2\alpha}\nu = m_{b\alpha}\frac{d^2x_{1\alpha}}{dt^2} + b_{m\alpha}\frac{d^2x_{2\alpha}}{dt^2}\nu,$$

Where f_α is the vector of force of labor of the α^{th} commodity. Obviously, through the above conversion, the vectors of forces of labor related to commodity quantity and quality are converted into those with equivalent units of measurement. In this case, the vector of force of labor spent for producing any commodity can serve as a homogenized measure of value.

Accordingly, dimensional analysis differs in theoretical forms for the needs of economic research, so as to make values in different dimensions satisfy the principle of dimensional homogeneity. To be specific, for value appreciation in commodity production, values in different commodity forms will be converted into the Lagrange functions of kinetic energy and potential energy [2]; for the measurement of commodity value in the form of statics, it will introduce the dimensionless method to establish an Euclidean metric space of commodity value and calculate the value of commodities through the scalar product [26]; for the measurement of commodity value in the form of dynamics, it will introduce the dimensionless processing method to establish a Riemannian metric space of commodity value and calculate the value of commodities through the scalar product in the tangent vector space on the base manifold of commodity value [27]; Finally, in a more general case, at a certain moment in the market circulation, the value of various commodities with certain material properties has been determined [22], so that under the principle of dimensional homogeneity, by dimensional analysis, the values of commodities can be calculated strictly and accurately.

2.3. Unit value of n commodities with dimensional homogeneity

The unit value of a commodity with dimensional homogeneity refers to the value embodied in the commodity that satisfies the value Euler equation [2] and the principle of dimensional homogeneity produced in a certain period of production, divided by its quantity and quality. For n commodities, the unit value can be defined as follows:

Definition 2.3.1. The value of a commodity can be expressed by the force of labor spent for production (labor value) and the force of labor compensated by consumption (use value). Thus there is:

$$w_\alpha = (w_{1\alpha}, w_{2\alpha}) = (f_{1\alpha}x_{1\alpha},\ f_{2\alpha}x_{2\alpha}) \in W^2_\alpha,$$ [6]

which is the vector of value of the α^{th} commodity, $\alpha = 1, 2, \ldots, n$, where $w_{1\alpha}$ is the commodity value related to quantity, and $w_{2\alpha}$ is the commodity value related to quality.

Here, commodity value has the dimension of energy $[L^2MT^{-2}]$. Although the value of any commodity has dimensionless units of measurement, no matter in the form of either quantity or quality, it should endow with a homogenized unit of measurement for the same commodity with the dimension of energy under dimensional homogeneity. Then for the α^{th} commodity, let the derived dimension from the dimensions of energy related to quantity and quality be

[5] In daily life, when we calculate the value of rice, both volume (bucket) and weight (kg) can be used as the material undertaker to measure the value of rice. Then, it is necessary to define the conversion coefficient between the volume (bucket) and weight (kg) for the same amount of value of rice under the principle of Dimensional Homogeneity. In this regard, for the dimension analysis on the value related to quantity and quality of the same commodity, it should follow the same logic.

[6] It means that the value of commodities is equal to $\bar{w}_{1\alpha}$ in Definition 2.3.2, based on the axiom that the value of commodities depends on the force of labor.

$$[L^2MT^{-2}]_{x_{1\alpha}}[LM^2T^{-2}]^{-1}_{x_{2\alpha}} = [LM^{-1}]_{(x_{1\alpha},x_{2\alpha})}, \quad (2.3.1)$$

and $[LM]^{-1}_{(x_{1\alpha},x_{2\alpha})} = \tau$, the sum of values related to quantity and quality of the α^{th} commodity can be expressed as

$$w_\alpha = w_{1\alpha} + w_{2\alpha}\tau = f_{1\alpha}x_{1\alpha} + f_{2\alpha}x_{2\alpha}\tau, \quad (2.3.2)$$

which is the value of the α^{th} commodity in the scalar form. Note that (2.3.1) and (2.3.2) have dimensional homogeneity, that is, since $L^2MT^{-2} \times LM^{-1} = LM^2T^{-2}$, the dimension of $f_{1\alpha}x_{1\alpha}$ is $[L^2MT^{-2}]_{x_{\alpha 1}}$, which is consistent with that of $f_{2\alpha}x_{2\alpha}\tau$.

Definition 2.3.2. Let $\bar{w}_\alpha$ be the unit value of the α^{th} commodity, then

$$\bar{w}_\alpha = \bar{w}_{1\alpha} + \bar{w}_{2\alpha}\tau, \quad (2.3.3)$$

where $\bar{w}_{1\alpha}$ is the unit value related to quantity, $\bar{w}_{2\alpha}$ is the unit value related to quality. From (2.3.2) and (2.3.3), $\bar{w}_{1\alpha}$ and $\bar{w}_{2\alpha}$ can be expressed as

$$\bar{w}_{1\alpha} = \frac{f_{1\alpha}x_{1\alpha}}{x_{1\alpha}} = \frac{w_{1\alpha}}{x_{1\alpha}} \text{ and } \bar{w}_{2\alpha} = \frac{f_{2\alpha}x_{2\alpha}}{x_{2\alpha}} = \frac{w_{2\alpha}}{x_{2\alpha}}.$$

Here (2.3.3) shows that the unit value of the α^{th} commodity is actually the work done by the force of labor for producing one unit quantity of the commodity, which can be converted from the work done by the force of labor for producing one unit quality[7] of the commodity through dimensionless coefficient. In this case, the dimensions of unit value of commodities still satisfy the principle of dimensional homogeneity for considering both quantity and quality at the same time.

2.4. Exchange value of commodities with dimensional homogeneity

In real life, the exchange of commodities is usually expressed by comparing the unit values of different commodities. For example, generally a gram of gold equals 50 kilograms of rice. Therefore, to study the commodity value in the process of market exchange, the exchange value will be the pivot, including the special and general equivalent forms of value. Obviously, on the premise that a certain commodity and any other commodities have the special equivalent form of value, all the special equivalent forms of value can aggregate the general equivalent form of value. [23] Therefore, in the general equivalent form of value, the exchange value of commodities refers to the ratio of the unit value of any commodity to the total value of all commodities in the whole society.

Definition 2.4.1. Let $N = \{\alpha \mid \alpha = 1, 2, \ldots, n\}$, $M = \{i \mid i = 1, 2, \ldots, n+1\}$, $N \subset M$, then the exchange value λ_α of the α^{th} commodity can be expressed as follows:

$$\frac{\bar{w}_\alpha}{\sum_{i=1}^{n+1} x_{i1}\bar{w}_{i1} + x_{i2}\bar{w}_{i2}\tau} = \lambda_\alpha. \quad (2.4.1)$$

Obviously, in (2.4.1), although with the same dimension of energy, there are still two different dimensional units of value, quantitative and qualitative, which will be homogenized by converting the derived dimension $[LM]^{-1}_{(x_{1\alpha},x_{2\alpha})} = \tau$ into the dimensional unit of quantity regarding the quality as a constant. For example, the commodities that meet the quality standards for trading in the market[8].

Therefore, for any α^{th} commodity, with the quality standard denoted as a constant of 1[9], then the dimensional conversion coefficient of unit value between the quantity of the α^{th} commodity and that of $(\alpha + i)^{th}$ commodity is

$$\frac{[L^2MT^{-2}]_{x_{1\alpha}}[LM^2T^{-2}]^{-1}_{x_{2\alpha}}}{[L^2MT^{-2}]_{x_{1i}}[LM^2T^{-2}]^{-1}_{x_{2i}}} = \frac{[LM]^{-1}_{(x_{1\alpha},\, x_{2\alpha})}}{[LM]^{-1}_{(x_{1i},\, x_{2i})}} = \varepsilon_{\alpha((x_{1\alpha},x_{2\alpha}),(x_{1\alpha+i},x_{2\alpha+i}),\alpha\neq i)}, \quad (2.4.2)$$

which is a dimensionless quantity. Let $\varsigma = ((x_{1\alpha}, x_{2\alpha}), (x_{1\alpha+i}, x_{2\alpha+i}))$, then $\varepsilon_{\alpha\varsigma}$ represents the conversion coefficient between the dimensional units of the quality of the α^{th} commodity and that of any $(\alpha + i)^{th}$ commodity.

[7] It is conventional that different commodities are expressed in different forms of unit quantity and unit quality at different times in different places. [23] The special forms of quantity and quality of various commodities are accidental, which yet do not deny their homogeneous attribute of being labor products. Therefore, a system of commensurable dimensions [2] can be established, where the unit quantity and unit quality of any commodity with different physical attributes can be compared and commensurated in the value category as the amount of products with standard quality through conversion coefficient.

[8] In civilized countries, this quality assurance is guaranteed by laws and regulations. In other words, the principle of dimensional homogeneity is often unconsciously established on the basis of human rational behaviors in the process of daily commodity exchange in the market.

[9] For example, taking 1g pure gold as the commodity with standard quality of 1, if the conversion coefficient of any commodity is 0.5, then the value of this commodity will be half of the value of 1g pure gold; if the value of a commodity is 300 times of the value of 1g pure gold, then its the conversion coefficient is 300, etc.

Therefore, when the quality standard is a constant of 1, the exchange value of the unit quantity of the α^{th} commodity can be expressed as:

$$\frac{w_{1\alpha}}{\sum_{i=1,i\neq\alpha}^{n+1} x_{1i}\bar{w}_{1i}\tau(1+\varepsilon_{\alpha\varsigma})} = \hat{\lambda}_{1\alpha}, \qquad (2.4.3)$$

which is the ratio of the value $\bar{w}_{1\alpha}$ of the unit quantity of the α^{th} commodity to the total value of all $n+1$ commodities, where the $(n+1)^{th}$ commodity refers to precious metal money as a measure of value[10].

2.5. Commodity price with dimensional homogeneity

The equilibrium price of a commodity refers to the product of its exchange value and the money supply[11] in a supply-demand balance, while the market price is the one when the market supply and demand are unbalanced [26]. When the quality is a constant, the joint function of equilibrium price and market price (hereafter as price) of the α^{th} commodity can be defined as follows:

Definition 2.5.1. According to Axiom 5 in the new theory of value [2], if the quality of any commodity is a constant, then the rational demand for this commodity will always has an upper boundary point, beyond which the extra supply will have no value, while the market price will rise in short supply [26]. Therefore, for the α^{th} commodity with standard quality 1, let p_α be the price of the α^{th} commodity[12], H be the set of money, $h \in H$ be the money supply[13], then its price will be

$$p_\alpha = \begin{cases} \dfrac{\dfrac{w_{1\alpha}}{\sum_{i=1,i\neq\alpha}^{n+1} x_{1i}\bar{w}_{1i}} h}{b_\alpha} = \dfrac{w_{1\alpha}}{\sum_{i=1,i\neq\alpha}^{n+1} x_{1i}\bar{w}_{1i}} \dfrac{b_\alpha^* h}{b_\alpha} = \dfrac{b_\alpha^* \lambda_\alpha h}{b_\alpha}, & b_\alpha^* \neq b_\alpha \\ \dfrac{w_{1\alpha}}{\sum_{i=1,i\neq\alpha}^{n+1} x_{1i}\bar{w}_{1i}} h = \lambda_\alpha h, & b_\alpha^* = b_\alpha, \end{cases} \qquad (2.5.1)$$

where $\bar{w}$, b_α, h are the independent variables of the commodity price function, $0 < b_\alpha < +\infty$, and the rational demand b_α^* is a constant coefficient.

The function indicates that, given the amounts of rational demand for various quality standards, the commodity price depends on three independent variables – unit value, production quantity and money supply by the following three properties:

(1) The price of every commodity is a piecewise function[14], which is equals to the ratio of its exchange value to the money supply in the balance of supply and demand, tends to zero as the supply tends towards infinity in oversupply, and tends to infinity as the supply tends towards zero in short supply.

(2) The price is in positive ratio to the money supply in the balance of supply and demand, while

[10] Generally speaking, this is so-called Marx's mathematical expression of commodity value with precious metal – gold – as the general equivalent form of value.

[11] In this paper, the functional relationship between money and value of commodities is assumed to be linear, under the premise of free flow of money within a closed economic system, where people's instinct of pursuing value maximization attracts money flow towards the commodity market with high purchasing power. Although changes in labor productivity always lead to changes in the unit value of different commodities, the spontaneous flow of money will quickly adjust the purchasing power of various commodity markets tend to be equilibrium.

[12] Here, the unit value of each commodity can be expressed as a certain price by being converted into the unit quantity of a commodity with a standard quality of 1, such as one gram of pure gold. Therefore, (2.5.1) shows the mathematical economic logic of Ricardo's quantity theory of money that between the value of commodities per unit quantity and the amount of money in circulation, there is a positive ratio, that is the price of commodities.

[13] The money discussed here is the one in an ideal state. That is to say, as value symbol, when it is precious metal money, the ratio of its value to the commodity value is the price; when it is a simple paper money, it brings a linear mapping between the value and price of the commodity, like mirror imaging, where the value is the original image, money is the mirror, and the price is the mirror image. In order to simplify the discussion, the velocity of money circulation is assumed to be constant. Therefore, the money supply here is a scalar. Clearly, the price formula for precious metal money also applies to paper money. Different from precious metal money, paper money is only a value symbol, without any value. Therefore, changes in the supply of paper money will lead to inflation or deflation, and change the distribution ratio of social wealth, which is more complicated, left for further discussion.

[14] Obviously, this conclusion has similarity with the law of diminishing marginal utility, yet differently, for the new theory of value, the diminishing marginal utility is just a phenomenon due to the market price deviating from the value of commodities particularly under oversupply, which happens to bring about overcapacity and overstock of commodities in the market. Nevertheless, marginalism regards the diminishing marginal utility as a basic axiom that as long as prices can be spontaneously adjusted, the market will never experience overproduction.

depends on the diminishing unit value and the money supply together in unbalanced supply and demand.

(3) The price is a convex function, where there must be a fixed point of equilibrium price [26].

3. Price equilibrium of commodities

After the discussion on value equilibrium of commodities, we further study the balance of market supply-demand for various commodities realized through the spontaneous regulation mechanism of price (hereafter price equilibrium). In this paper, price equilibrium refers to how to maximize of the commodity value and at the same time maintain the market balance between supply and demand by adjusting the quantity of each product under the market price mechanism in commodity exchange, given the rational demand for each product and all commodities circulating in the market satisfying the provisions of value equilibrium in a process of production and circulation of various commodities? Here, it is assumed that the value and price of any commodity discussed below has completed the dimensional analysis and established homogeneous dimensions, that is, satisfying the value and price of commodities defined in (2.5.1).

3.1. Stability of mapping and zero solution to equations of commodity price

First, we examine the mathematical definition of economic concepts related to the stability of market price of n commodities. In order to simplify the discussion, we focus on the price of commodities with constant quality and the equilibrium price on the hyperplane of commodity price.

Definition 3.1.1. If the quality of commodities is constant quantity, and the price of commodities is the product of the exchange value of commodity per unit quantity and the money supply, let $B^n \times H \mapsto P^n$ be the mapping of hyperplane of $n+1$-dimensional vector space of commodity and money with homogenized value dimension, where $b=(b_1, b_2, \ldots, b_n) \in B^n$ is the vector in the vector space of commodity quantity with homogenized dimension, $h \in H$ is the money supply expressed by one-dimensional real numbers, given the unit value of each commodity $\bar{w}=(\bar{w}_1, \bar{w}_2, \ldots, \bar{w}_n)$, the rational demand $b^*=(b_1^*, b_2^*, \ldots, b_n^*)$ and the money supply H, then the mapping of commodity price will be expressed as

$$p : B^n \times H \mapsto P^n$$

$$(b_1, b_2, \cdots, b_n, h) \mapsto (p_1, p_2, \cdots, p_n),$$

where P^n is called the n-dimensional Euclidean space of commodity price with constant quality, and $p(b, h)$ is the mapping of commodity price, that is

$$\begin{aligned} p(b, h) &= (p_1(b), p_2(b), \cdots, p_n(b)) \\ &= \left(\frac{\bar{w}_1}{\sum_{\alpha=1}^{n} b_\alpha^* \bar{w}_\alpha} \frac{b_1^*}{b_1} h, \frac{\bar{w}_2}{\sum_{\alpha=1}^{n-1} b_\alpha^* \bar{w}_\alpha} \frac{b_2^*}{b_2} h, \cdots, \frac{\bar{w}_n}{\sum_{\alpha=1}^{n} b_\alpha^* \bar{w}_\alpha} \frac{b_n^*}{b_n} h \right) \\ &= (\lambda_1, \lambda_2, \cdots, \lambda_n) \\ &= \lambda. \end{aligned}$$

In particular, under the condition of supply-demand balance, $b_\alpha = b_\alpha^*$, let $x_\alpha = b_\alpha^*$, the mapping of commodity equilibrium price is expressed as

$$\begin{aligned} p^*(b) &= (p_1^*(b), p_2^*(b), \cdots p_n^*(b)) \\ &= \left(\frac{\bar{w}_1}{\sum_{\alpha=1}^{n} b_\alpha^* \bar{w}_\alpha} \frac{b_1^*}{x_1} h, \frac{\bar{w}_2}{\sum_{\alpha=1}^{n} b_\alpha^* \bar{w}_\alpha} \frac{b_2^*}{x_2} h, \cdots, \frac{\bar{w}_n}{\sum_{\alpha=1}^{n} b_\alpha^* \bar{w}_\alpha} \frac{b_n^*}{x_n} h \right) \\ &= (p_1^*, p_2^*, \cdots, p_n^*) \qquad (3.1.1) \\ &= p^*, \end{aligned}$$

which is the equilibrium price of commodities.

Definition 3.1.2. The function of commodity price difference refers to the difference between the market price and the equilibrium price (3.1.1) of commodities, that is

$$\Delta p_\alpha = p_\alpha(b) - p_\alpha^*(b), \ \alpha = 1, 2, \ldots, n,$$

then $\forall p(b) \in U \subset P^n$, U is a bounded closed set, let $y = b_1^* \bar{w}_1 + b_2^* \bar{w}_2 + \cdots b_n^* \bar{w}_n$, if it satisfies the differential equation of commodity price difference

$$\frac{d\Delta p_\alpha}{dt} = \frac{dp_\alpha}{dt} - \frac{dp_\alpha^*}{dt} = 0, \quad \alpha = 1, 2, \ldots, n-1, \qquad (3.1.2)$$

then

$$\frac{dp_\alpha}{dt} = \frac{d}{dt}\left(\bar{w}_\alpha \frac{b_\alpha^*}{b_\alpha} yh \right) = \bar{w}_\alpha \frac{-b_\alpha^*}{b_\alpha^2} yh, \ \ \alpha = 1, 2, \ldots, n,$$

$$\frac{dp_\alpha^*}{db} = \frac{d}{db}\left(\bar{w}_\alpha \frac{-b_\alpha^*}{x_\alpha} yh \right) = \bar{w}_\alpha \frac{-b_\alpha^*}{x_\alpha^2} yh, \ \ \alpha = 1, 2, \ldots, n,$$

$$\text{Let} \left(\frac{-\bar{w}_\alpha b_\alpha^*}{b_\alpha^2} yh \right) - \left(\frac{-\bar{w}_\alpha b_\alpha^*}{x_\alpha^2} yh \right) = 0, \ \max \ x_\alpha = b_\alpha = b_\alpha^*,$$

here U is a bounded closed set in the Euclidean space of commodity price corresponding to n-dimensional commodity quantity under equivalent exchange. Then, the differential equation of commodity price defined in the bounded closed set U is called the equation of commodity equivalent exchange satisfying the constraints of exchange value.

Definition 3.1.3. Let U be a bounded closed set in the Euclidean space of commodity price corresponding to n-dimensional commodity quantity satisfying equivalent exchange, if the value function of any α^{th} commodity in (3.1.2) has the properties of differential equation of value equilibrium, that is the differential equation (3.1.2) with implicit solution (3.1.3), then $\forall p(b) \in U \subset P^n$, there is

$$\begin{cases} \left\{ \dfrac{dp_\alpha}{dt} = \dfrac{d}{dt}(p(b_\alpha)) = \dfrac{d}{dt}\left(\dfrac{\bar{w}_\alpha}{\sum\limits_{\alpha=1}^{n-1} b_\alpha^* \bar{w}_\alpha} \dfrac{b_\alpha^*}{b_\alpha} \right) = 0 \right. \\ \begin{cases} \dfrac{dw_\alpha}{dt} = \dfrac{d}{dt}(w(f_{1\alpha}, f_{2\alpha})) \\ = \dfrac{d}{dt}(M(f_{1\alpha}, f_{2\alpha})df_{1\alpha} - N(f_{1\alpha}, f_{2\alpha})df_{2\alpha}) \\ = \dfrac{\partial w}{\partial f_{1\alpha}} df_{1\alpha} + \dfrac{\partial w}{\partial f_{2\alpha}} df_{\alpha 2} \\ = 0, \end{cases} \end{cases} \quad (3.1.3)$$

$$\alpha = 1, 2, \ldots, n$$

which is called the n-dimensional differential equation of commodity price corresponding to quantity satisfying the constraints of exchange value.

Definition 3.1.4. Let $\forall p(b)$ be a differential equation of n-dimensional commodity price defined on U and satisfying (3.1.3), if $p(b)$ is also the one with global stability, then $p(b)$ is a differential equation of n-dimensional commodity price satisfying the law of exchange value. In particular, if $p(b)$ defined on U satisfies (3.1.3) and global stability, then U is the region where the law of exchange value is valid.

Here, we define the differential equation with global stability as below: for the vector differential equation $p(b)$ of any α^{th} commodity price, $\alpha = 1, 2, \ldots, n$, given $\varepsilon > 0$, there is $\delta > 0$ (generally related to ε and t_0), when any $b_{\alpha 0}$ satisfies $\|b_{\alpha 0}\| \le \delta$, the solution $b(t)$ determined by the initial value condition $b_\alpha(t_0) = b_{\alpha 0}$ of the equations (3.1.3), for all $t \ge t_0$, there always be

$$\|b(t)\| < \varepsilon,$$

then the zero solution $\|b_{\alpha 0}\| = 0$ to (3.1.3) is stable, where $\|b\|$ is the scalar product of commodity value.

If the zero solution $b_\alpha = 0$ to (3.1.3) is stable, and there is $\delta_0 > 0$, so that when $\|b_0\| \le \delta_0$, the solution b_α satisfying the initial value condition $b_\alpha(t_0) = b_{\alpha 0}$ will be

$$\lim_{t \to +\infty} b(t) = 0, \ \alpha = 1, 2, \ldots, n, \quad (3.1.4)$$

then the zero solution $b_\alpha = 0$ is asymptotically stable. Therefore, if the vector ordinary differential equation $b(t)$ of commodity unit value defined in U satisfies the conditions (3.1.3) and (3.1.4), then U is the region where the law of exchange value is valid.

3.2. Law of exchange value[15]

The law of exchange value refers to the economic logic relationship satisfying the conditions of (3.1.3) and (3.1.4). Specifically, in the real commodity economy system, the law of exchange value is represented through the instinctive and spontaneous economic behaviors by a large population of individuals with intelligence and subjective initiative in the process of market transactions, which can be illustrated as below:

In a certain industry, a certain kinds of products are spontaneously produced with certain quality assumed as constant, the unit value $\bar{w}_\alpha$ and the rational demand b_α^* of the product are known, while whether the total production quantity aggregated by various producers in the industry is consistent with the rational demand is unknown until a certain period of production completes. That is, if all producers are seeking to maximize the value of individual wealth, according to the formula of commodity price, it is necessary to minimize the value difference Δw_α of the α^{th} product, mathematically expressed as follows:

$$\Delta w_\alpha = w(p_\alpha) = b_\alpha p_\alpha - b_\alpha p_\alpha^*, \quad (3.2.1)$$

which indicates that the value difference Δw_α of the α^{th} product depends on the product of the market price and the actual production quantity minus the product of the equilibrium price and the actual production quantity, where $b_\alpha p_\alpha$ is the product of the actual market price and the actual production quantity of the commodity, $b_\alpha p_\alpha^*$ is the product of the equilibrium price and the actual production quantity.

Obviously, in order to make the production plan, each producer should know the actual market demand quantity $D = b_\alpha - b_\alpha^*$. After countless trials and errors, the vast majority of producers in a mature market have recognized the law of value, that is, to expand production if making profits, and to reduce production if losing money, so that the actual production quantity will close to the rational demand quantity. Therefore, the actual market demand quantity D is a function of the real price p_α of commodities,

[15] It is actually a strict mathematical expression of Smith's "invisible hand" [28] and Marx's "law of value" [25]. According to Smith [28], "*by directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention. Nor is it always the worse for the society that it was no part of it. By pursuing his own interest he frequently promotes that of the society more effectually than when he really intends to promote it*".

$$D_\alpha = D(p_\alpha). \qquad (3.2.2)$$

Clearly, the supply quantity Q_α in the market should be equal to the actual demand quantity D, that is

$$Q_\alpha = D(p_\alpha), \qquad (3.2.3)$$

then, substitute (3.2.2) and (3.2.3) into (3.2.1), and get

$$\Delta w_\alpha = w(p_\alpha) = p_\alpha D(p_\alpha) - p_\alpha^* D(p_\alpha). \quad (3.2.4)$$

According to the extremum solution theory of calculus, the market price p can be obtained from the following equation when the price difference Δw is minimum

$$\frac{dw(p_\alpha)}{dp_\alpha} = 0. \qquad (3.2.5)$$

In real economic management activities, if producers want to achieve the maximum value, they must "know themselves and know their opponents". However, the real market economy is a spontaneous production process, where most producers only "know themselves", but know few about their opponents. In this case, can they find an effective way to realize the market price when the value is maximum? The answer is YES. Now we analyze this production strategy.

Assuming that every producer pursues value maximization, the market regulation mechanism will force every producer to obey the law of value: producing the same quantity will reduce the revenue or even suffer a loss in oversupply, and obtain excess value in short supply. Therefore, in the process of commodity production, in order to pursue profit maximization, the production quantity should be reduced in oversupply, and increased in short supply. Then, the supply and demand of every commodity will return to balance. That is, under the law of value, the market regulation mechanism roughly can be divided into four situations:

(1) To increase the price if both the price and the revenue increase.

Supposing at the t-th period, $p_\alpha(t)$ is the price of a product with a revenue $w_\alpha(t)$. After a certain period Δt, the price of the product changes to $p_\alpha(t+\Delta t)$, and the revenue changes to $w_\alpha(t+\Delta t)$. In this case, the price increases, $p_\alpha(t+\Delta t) - p_\alpha(t) > 0$, the revenue also increases, $w_\alpha(t+\Delta t) - w_\alpha(t) > 0$. Then continue to increase the price at one more period Δt, $p_\alpha(t+2\Delta t) > p_\alpha(t+\Delta t)$, if the change in price per unit time $[p_\alpha(t+\Delta t) - p_\alpha(t)]/\Delta t$ has direct ratio with the change in revenue per unit price $[w_\alpha(t+\Delta t) - w_\alpha(t)]/[p_\alpha(t+\Delta t) - p_\alpha(t)]$, there will be

$$\frac{p_\alpha(t+\Delta t) - p_\alpha(t)}{\Delta t} = \gamma \frac{w_\alpha(t+\Delta t) - w_\alpha(t)}{p_\alpha(t+\Delta t) - p_\alpha(t)}, \quad (3.2.6)$$

Where $\gamma > 0$. When Δt is small enough, the above equation will be

$$\frac{dp_\alpha}{dt} = \gamma \frac{\partial w_\alpha}{\partial p_\alpha} \qquad (3.2.7)$$

where the partial derivatives is used to show that the change in revenue is only caused by the change in price.

(2) To decrease the price if the price increases, while the revenue decreases.

In this case, the price increases, $p_\alpha(t+\Delta t) > p_\alpha(t)$, the revenue decreases, $w_\alpha(t+\Delta t) < w_\alpha(t)$, and then $p_\alpha(t+2\Delta t) < p_\alpha(t+\Delta t)$. If the change in price per unit time has direct ratio with the change in revenue per unit price, the corresponding mathematical model will be the same with (3.2.6).

If $\partial w_\alpha / \partial p_\alpha < 0$, then the price should be decreased $\mathrm{d}p_\alpha / \mathrm{d}t < 0$, and the change in price is in line with the same ratio, i.e. $\mathrm{d}p_\alpha / \mathrm{d}t = \gamma\, \partial w_\alpha / \partial p_\alpha$.

(3) To decrease the price if the price decreases, while the revenue increases.

As the price decreases, $\Delta p_\alpha < 0$, the revenue increases, $w_\alpha > 0$, i.e. $\partial w_\alpha / \partial p_\alpha < 0$, then the decrease in price should be continued, $\mathrm{d}p_\alpha / \mathrm{d}t < 0$, the mathematical model for price regulation will be the same with (3.2.6) and (3.2.7).

(4) To increase the price if both the price and the revenue decrease.

As the price decreases, $\Delta p_\alpha < 0$, the revenue decreases, $\Delta w_\alpha < 0$, i.e. $\partial w_\alpha / \partial p_\alpha > 0$, then the price should be increased, $\mathrm{d}p_\alpha / \mathrm{d}t > 0$, the mathematical model for price regulation will be the same with (3.2.6) and (3.2.7).

The above strategies for market regulation mechanism is called the function $w(p_\alpha)$ of commodity exchange value[16].

[16] Through the above analysis, we can see that the function of commodity exchange value has the property of diminishing marginal revenue. On the surface, this conclusion is consistent with the law of diminishing marginal utility by marginalism. However, this is not the case. In terms of economic meaning, there is fundamental difference that the diminishing marginal revenue of commodity exchange value is one important theorem deduced from the basic axiomatic system in the new theory of value, which shows a special case of the law of value when commodity oversupply in the market, but regarded as a basic axiom, and used to explain all economic phenomena by marginalism. Consequently, the theory of marginal utility has the error of taking a part for the whole, failing to explain all kinds of disequilibrium phenomena.

Definition 3.2.1. Let U be a bounded closed set in the Euclidean space of commodity price corresponding to n-dimensional quantity under barter exchange, if the differential functions of commodity value defined in U satisfy the constraints of exchange value, that is, $w(p_\alpha)$ is a concave function, objectively there is an equilibrium price p^* so that

$$\lim_{t\to\infty} p_\alpha(t) = p_\alpha^*,$$

then the function $w(p_\alpha)$ of commodity exchange value that plays a dominant role in the regulation mechanism for supply-demand balance in the process of commodity production is called the function of commodity value that satisfies the constraints of the law of exchange value.

Theorem 3.1. Let U be a bounded closed set in the Euclidean space of commodity price corresponding to n-dimensional quantity under equivalent exchange, if the law of exchange value plays a dominant role in U, for the function $p(t)$ of price of any α^{th} commodity, there must be a maximum, when

$$\frac{dp_\alpha}{dt} = 0,$$

so that the market supply and demand of the commodity are in a balance, that is

$$b(t) = \frac{db}{dt} = v, \ vt = b_\alpha^*.$$

Proof: If $w(p_\alpha)$ is a concave function, there is an equilibrium price λ^* to minimize the value difference Δw, then according to the above pricing strategy, so that

$$\lim_{t\to\infty} p_\alpha(t) = p_\alpha^*. \qquad (3.2.8)$$

Define a Lyapunov energy function $V(t)$ of value

$$V_\alpha(t) = \frac{[p_\alpha(t) - p_\alpha^*]^2}{2} \geq 0. \qquad (3.2.9)$$

Let $V(t)$ be derived by time t, and put (3.2.7), i.e. $\mathrm{d}p_\alpha/\mathrm{d}t = \gamma(\partial w_\alpha/\partial p_\alpha)$ into (3.2.9), then

$$\frac{dV}{dt} = (p(t) - p^*)\gamma\frac{\partial w}{\partial p}. \qquad (3.2.10)$$

According to (3.1.2):

$$\frac{dp_\alpha}{dt} = \frac{d}{dt}\left(\bar{w}_\alpha \frac{b_\alpha^*}{b_\alpha} yh\right) = \bar{w}_\alpha \frac{-b_\alpha^*}{b_\alpha^2} yh, \quad \alpha = 1,2,\ldots,n,$$

$$\frac{dp_\alpha^*}{dt} = \frac{d}{dt}\left(\bar{w}_\alpha \frac{b_\alpha^*}{x_\alpha} yh\right) = \bar{w}_\alpha \frac{-b_\alpha^*}{x_\alpha} yh, \quad \alpha = 1,2,\ldots,n,$$

Let $\left(\frac{-\bar{w}_\alpha b_\alpha^*}{b_\alpha^2} yh\right) - \left(\frac{-\bar{w}_\alpha b_\alpha^*}{x_\alpha^2} yh\right) = 0, \ \max \ x_\alpha = b_\alpha = b_\alpha^*,$

when $b_\alpha = x_\alpha = b_\alpha^*$,

$$\frac{dV_\alpha}{dt} = \frac{\partial V_\alpha}{\partial p_\alpha}\frac{dp_\alpha}{dt}$$
$$= (p_\alpha(t) - p_\alpha^*)\frac{dp_\alpha}{dt} \qquad (3.2.11)$$
$$= (\bar{w}_\alpha \frac{b_\alpha^*}{b_\alpha} yh - \bar{w}_\alpha \frac{-b_\alpha^*}{x_\alpha} yh)\bar{w}_\alpha \frac{-b_\alpha^*}{b_\alpha^2} yh$$

then

$$\lim_{t\to\infty} p_\alpha(t) = p_\alpha^*.$$

The commodity exchange market is an economic system dominated by the law of exchange value[17].

Furthermore, if there are n types of products in the whole society, where the product prices are $p_1, \ldots, p_n$ respectively, then the value of each product is the function of its price respectively,

$$w(p_1), w(p_2), \ldots, w(p_n).$$

Then it is assumed that the production of each product is linearly independent[18], when $\forall w(p_\alpha)$, $\alpha = 1, 2, \ldots, n$, from (3.2.11), there must be a corresponding optimal product supply quantity b_α^* to maximize the revenue $w(p_\alpha)$. By the above production strategy, there must be

$$\lim_{t\to\infty} p_\alpha(t) = p_\alpha^*, \ \alpha = 1, 2, \ldots, n$$

and a balance between supply and demand for all commodities

$$\lim_{t\to\infty}(b_\alpha(t) - b_\alpha^*) = 0, \ \alpha = 1, 2, \ldots, n$$

4. Conclusion

The law of exchange value of commodities represents human's pursuit of an ideal state. In real life, the pursuit of ideal is a kind of spontaneous and incompletely rational behaviors with a large number of blind spots by accessing to local information. The general expression of these behaviors is a piecewise

[17] The economic system governed by the law of exchange value does not exist forever. On the contrary, it is the product of modern human civilization. Here, the political foundation for market economy lies in legal systems such as free trade, monetary and etc., without which, in an era full of violence, "Free Trade and Equivalence Exchange" of the market economy can only be an illusion in such an era of violence under the law of the jungle.

[18] The situation will be more complicated if the productions of different products are linearly related, where it is necessary to figure out the solution to differential equations of multivariate functions. Interested readers can discuss it by themselves.

function: in the state of supply-demand balance, the price of commodities is dominated by the exchange value; in the state of unbalanced supply and demand, the price deviates from the value. Nevertheless, in the human society governed by rational people, the vast majority follow objective economic laws, pursue the maximization of individual interests, so that the supply and demand of commodities are balanced in the market, that is, the law of exchange value has become a force that dominates human society, which is the so-called "invisible hand", guiding mankind to pursue the ideal state of supply-demand balance. In this case, the unit exchange value of each commodity is the ratio of its unit value to the total value of all commodities in the society, where it is assumed that every rational person conducts his economic behaviors according to the law of exchange value.

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Information about authors

Jie Wu – Researcher, Institute of Shandong Development, Shandong University, 27 Shanda Road, Jinan 250100, China; Guangzhou Milestone Software Co., Ltd. (Economic Simulation Base of the National Simulation and Control Engineering Research Center), 10B, Tower A, Guangdong International Building, 339 Huanshi Dong Road, Yuexiu District, Guangzhou 510098, China; e-mail: jw@gzmss.com

Zili Wu – Engineer, Guangzhou Milestone Software Co., Ltd. (Economic Simulation Base of the National Simulation and Control Engineering Research Center), 10B, Tower A, Guangdong International Building, 339 Huanshi Dong Road, Yuexiu District, Guangzhou 510098, China; e-mail: wzl@gzmss.com

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