Forecasting Realized Volatility of Russian stocks using Google Trends and Implied Volatility

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Abstract. This work proposes to forecast the Realized Volatility (RV) and the Value-at-Risk (VaR) of the most liquid Russian stocks using GARCH, ARFIMA and HAR models, including both the implied volatility computed from options prices and Google Trends data. The in-sample analysis showed that only the implied volatility had a significant effect on the realized volatility across most stocks and estimated models, whereas Google Trends did not have any significant effect. The out-of-sample analysis highlighted that models including the implied volatility improved their forecasting performances, whereas models including internet search activity worsened their performances in several cases. Moreover, simple HAR and ARFIMA models without additional regressors often reported the best forecasts for the daily realized volatility and for the daily Value-at-Risk at the 1 % probability level, thus showing that efficiency gains more than compensate any possible model misspecifications and parameters biases. Our empirical evidence shows that, in the case of Russian stocks, Google Trends does not capture any additional information already included in the implied volatility.

Keywords: forecasting, realized volatility, value-at-risk, implied volatility, google trends, GARCH, ARFIMA, HAR

Introduction

Volatility forecasting is of cardinal importance in several applications, from derivatives pricing to portfolio and risk management, see Bauwens et al. [1] for a large survey. Recent literature suggested the idea to consider the investors' behavior measured by the internet search volumes as a factor influencing the assets volatility, see for example Campos et al. [2] and references therein for more details. The investors' interest was originally quantified using some proxy measures like news or turnover. However, Donaldson and Kamstra [3] showed that these proxies do not improve the forecasting of volatility. Instead, recent works by Andrei and Hasler [4] and Vlastakis and Markellos [5] reported empirical evidence showing that online search volumes are a good predictor of volatility.

This paper aims to estimate the predictive power of online search activity (as proxied by Google Trends data) and implied volatility (computed from option prices) for forecasting the realized volatility of several Russian stocks. In this regard, the implied volatility measures the investors' sentiment about the future performance of an asset, see the survey of Mayhew [6] and references therein for more details. These two measures of investors' attention and expectations are then used to forecast the realized volatility of Russian stocks by using three competing models: the Heterogeneous Auto-Regressive (HAR) model by Corsi [7], the AutoRegressive Fractional (ARFIMA) model by Andersen et al. [8], and a simple GARCH(1,1) model. The forecasting performances of these models are compared using forecasting diagnostics such as the mean squared error (MSE), and the Model Confidence Set by Hansen et al. [9]. The models' volatility forecasts are also employed to compute the Value-at-Risk (VaR) for each asset to measure their market risk.

The first contribution of this paper is an evaluation of the contribution of both online search intensity and options-based implied volatility to the modelling of realized volatility for Russian stocks. To our knowledge, this analysis has not been done elsewhere. The second contribution is an out-of-sample forecasting exercise of realized volatility using several alternative models specifications, with and without Google data and implied volatility. The third contribution of the paper is a backtesting exercise to measure the accuracy of Value-at-Risk forecasts.

Literature review

There is an increasing body of the financial literature which examines how online searches affect asset pricing and volatility modelling.

Vlastakis and Markellos [5] considered the top-30 stocks (in terms of volume) traded on the NYSE and used the search volumes involving the name of the company as a proxy of demand for firm-specific information. They found that such demand for information contains potentially useful signals because it is strongly related to the stock trading volumes and the historical volatility. Campos and Cortazar [2] evaluated the marginal predictive power of Google trends to forecast the Crude Oil Volatility

index by using HAR models and several macro-finance variables. More specifically, they employed the standard HAR model, the HAR model including macroeconomic variables, the HAR model with online search volumes and the HAR model including both search volumes and macroeconomic variables. They found that the amount of online searches has a positive relationship with the oil volatility index. Moreover, this association remains significant even when macroeconomic variables are included in the model, thus highlighting that Google data capture some extra information.

Goddard and Wang [9] examined the relationship between investors' interest and the foreign exchange market volatility. They showed a strong connection between the changes in volatility and the changes in online attention, even after controlling for macroeconomic variables. Basistha et al. [10] evaluated the role of the online search activity for forecasting realized volatility of financial markets and commodity markets using models that also include market-based variables. They found that Google search data play a minor role in predicting the realized volatility once implied volatility is included in the set of regressors. Therefore, they suggested that there might exist a common component between implied volatility and Internet search activity: in this regard, they found that most of the predictive information about realized volatility contained in Google Trends data is also included in implied volatility, whereas implied volatility has additional predictive content that is not captured by Google data.

Methodology
A. Measures of volatility
Realized Variance. Real volatility is not observable, so a proxy is needed for its observation. The realized variance **(RV)** is probably the best proxy available: Barndorff–Nielsen and Shephard [11] showed that the RV is a consistent estimator of the actual variance, while Liu et al. [12] compared more than 400 estimators of price variation and they came to the conclusion that it is difficult to significantly outperform the 5-minute RV estimator. For this reason, we used this estimator in this work. Suppose that on the trading day t, M prices were observed at times t_0, t_1, ..., t_M. If p_{tj} stands for the logarithmic price at time t_j, then the log-return r_{tj} for the j-th interval of day t is defined as, $r_{tj} = p_{tj} - p_{tj-1}$. The formula for the realized variance is thus given by

$$\mathrm{RV}_t = \sum_{j=1}^{M} r_{tj}^2.$$

Over a time horizon of k days, the realized variance is computed as $\mathrm{RV}_{t:t+k} = \Sigma_{i=1}^{k} \mathrm{RV}_{t+i}$, under the convention that $\mathrm{RV}_t = \mathrm{RV}_{t-1:t}$. Horizons of 1 (daily), 5 (weekly) and 22 (monthly) days were considered.

Implied volatility. The implied volatility **(IV)** of an option contract is the value of the volatility of the underlying asset which makes the theoretical value of the option – computed using an option pricing model like the Black–Scholes model-equal to the current market price of the option, see Mayhew [6] and Hull [13] for details. The implied volatility reflects the investors'

expectations and sentiments and, if the assumptions of the Black–Scholes model hold, it is an efficient predictor of the actual volatility of the underlying asset. Assuming that all the other parameters of the Black-Scholes model are available (that is, the stock and strike prices, the risk-free rate, the time to expiration and the market price of the option), then the IV can be computed using nonlinear optimization methods, like the Newton–Raphson algorithm, see Fengler [14] and references therein.

B. Data
Intraday data sampled every 5 minutes for the most liquid Russian stocks (Sberbank – *SBER*, Rosneft – *ROSN*, Yandex – *YNDX*, Gazprom – *GAZP*, where the four-letter abbreviations are the stocks tickers) were downloaded from the website *finam.ru*. Option prices from the Moscow exchange were used to compute the implied volatility for each asset. The dataset covered the period from January 2016 till April 2018.

Google Trends computes how many searches were made for a keyword or a topic on Google over a specific period of time and a specific region. This amount is then divided by the total amount of searches for the same period and region, and the resulting time series is divided by its highest value and multiplied by 100, so that all data are normalized between 0 and 100. The tickers of the Russian stocks described above were used as a search keyword used by investors to get information for a particular company. All search volumes were downloaded from the Google Trends website using the R package «*gtrendsR*». These data were then merged with the search volumes for the queries in Russian looking for a specific stock price, for example «Sberbank share price».

C. Models
HAR model. The Heterogeneous Auto-Regressive model for the realized volatility was first proposed by Corsi [7] and it allows to reproduce several stylized facts of assets' volatility. The HAR model is specified as follows,

$$\mathrm{RV}_{t+1} = \beta_0 + \beta_D \mathrm{RV}_t + \beta_W \mathrm{RV}_{t\text{-}5:t} + \beta_M \mathrm{RV}_{t\text{-}22:t} + \varepsilon_{t+1},$$

where D, W and M stand for daily, weekly and monthly values of the realized volatility, respectively. The main novelty of our work is the inclusion of the implied volatility and Google Trends as additional regressors to forecast the realized volatility of Russian stocks:

$$\mathrm{RV}_{t+1} = \beta_0 + \beta_D \mathrm{RV}_t + \beta_W \mathrm{RV}_{t\text{-}5:t} + \beta_M \mathrm{RV}_{t\text{-}22:t} + \\ + \gamma \mathrm{GT}_t + \alpha \mathrm{IV}_t + \varepsilon_{t+1}$$

ARFIMA model. Andersen et al. [8] proposed the Auto-Regressive Fractional Integrated Moving Average **(ARFIMA)** model to forecast the realized volatility, and it has been one of the best models ever since. The ARFIMA(p,d,q) model is given by:

$$\Phi(L)(1 - L)^d (\mathrm{RV}_{t+1} - \mu) = \Theta(L)\varepsilon_{t+1},$$

where L is the lag operator, $\Phi(L) = 1 - \varphi_1 L - \ldots - \varphi_p L^p$, $\Theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$ and $(1 - L)^d$ is the fractional differencing operator defined by

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)},$$

where $\Gamma(\cdot)$ is the gamma function. Similarly to the HAR model, we also considered the case where the implied volatility and Google Trends are added as (external) regressors, so the model becomes

$$\Phi(L)(1-L)^d(RV_{t+1} - \mu) = \gamma GT_t + \alpha IV_t + \Theta(L)\varepsilon_{t+1}.$$

Hyndman and Khandakar [15] proposed an algorithm for the automatic selection of the optimal ARFIMA model, which is implemented in the R packages *forecast* and *rugarch*.

GARCH model. The Generalized Auto-Regressive Conditional Heteroscedasticity **(GARCH)** models are extensively used in empirical finance, thanks to their good forecasting performances: for example, Hansen and Lunde [16] compared more than 330 volatility models and they found no evidence that a GARCH(1,1) can be outperformed by more sophisticated models. A general GARCH(p,q) model for the conditional variance equation can be specified as follows

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2,$$

where σ_t^2 is the conditional variance at time t. A simple GARCH (1,1) model with standardized errors following a Student's t-distribution was employed in this work. Similarly to the HAR and ARFIMA models, we also considered a GARCH specification including the implied volatility and Google Trends as additional regressors.

D. Forecast Comparison

Variance forecasts. We divided the data into a training dataset used to estimate the models (the first 67 % of the sample), and a test dataset to evaluate the models' performances. We computed the 1-day-ahead volatility forecasts of our competing models and we compared them using the mean square error **(MSE)**. We also employed the Model Confidence Set proposed by Hansen et al. [17], which can select the best performing model(s) at a predefined confidence level. Given a specific loss function, in our case the squared loss, $\text{Loss}_t = \left(\widehat{RV}_i - RV_i\right)^2$, where RV and $\widehat{RV}$ stand for the observed and forecasted level of the realized volatility, then the difference between the losses of models i and j at time t can be computed as follows

$$d_{i,j,t} = \text{Loss}_{i,t} - \text{Loss}_{j,t}.$$

The hypothesis of equal predictive ability suggested by Hansen et al. [17] can be formulated as

$$H_{0,M} : E(d_{i,j,t}) = 0, \text{ for all } i,j \in M,$$

where M is the set of forecasting models. First, the following t-statistics are calculated

$$t_{ij} = \frac{\overline{d}_{ij}}{\sqrt{\widehat{\text{var}}(\overline{d}_{ij})}} \text{ for } i,j \in M,$$

where $\overline{d}_{ij} = T^{-1}\Sigma_{t=1}^T d_{ij,t}$, and $\widehat{\text{var}}(\overline{d}_{ij})$ is a bootstrapped estimate of $\text{var}(\overline{d}_{ij})$. Then, this test statistic is computed: $T_{R,M} = \max_{i \in M} |t_{ij}|$. This statistic has a non-standard distribution, so the distribution under the null hypothesis is computed via bootstrap. If the null hypothesis is rejected, one model is eliminated from the analysis by using the following rule

$$e_{RM} = \arg\max_i \left\{ \sup_{j \in M} \frac{\overline{d}_{ij}}{\sqrt{\widehat{\text{var}}(\overline{d}_{ij})}} \right\},$$

The number of models is diminished by 1 and the testing procedure starts from the beginning. For volatility forecasts, the previous MSE loss was used, whereas the symmetric quantile loss function proposed by Gonzalez–Rivera et al. [18] was used for the VaR forecasts (more details in the next section).

Value-at-Risk forecasts. The Value-at-Risk (VaR) is the potential market loss of a financial asset over a time horizon h with probability level α. The VaR is a widely used measure of market risk in the financial sector, and we refer to Louzis et al. [19] for a large survey of realized volatility models and VaR methods.

In this work, we considered $h = 1$ and the probability levels $\alpha = 5$ % and $\alpha = 1$ %. In the case of HAR and ARFIMA models, the 1-day ahead VaR can be computed as follows

$$\text{VaR}_{t+1,\alpha} = \Phi_\alpha^{-1}\sqrt{\widehat{RV}_{t+1}}$$

where Φ_α^{-1} is the inverse function of a standard normal distribution function at the probability level α, while $\widehat{RV}_{t+1}$ is the 1-day-ahead forecast for the realized volatility. In the case of GARCH models with standardized student's t errors, the 1-day ahead VaR can be computed as follows

$$\text{VaR}_{t+1,\alpha} = \hat{\mu}_{t+1} + t_{\alpha,\upsilon}^{-1}\sqrt{\hat{\sigma}_{t+1}^2}$$

where $\hat{\mu}_{t+1}$ is the 1-day-ahead forecast of the conditional mean, $\hat{\sigma}_{t+1}^2$ is the 1-day-ahead forecast of the conditional variance, while $t_{\alpha,\upsilon}^{-1}$ is the inverse function of the standardized Student's t distribution with υ degrees of freedom at the probability level α.

To compare the forecasting performance of the different VaR models, the forecasted values of the VaR are compared to the actual returns for each day, and the number of times when the ex-ante forecasted VaR is smaller than the actual loss is counted (that is, the number of violations are counted): a "perfect VaR model" would deliver a number of violations which is not predictable and exactly equal to α (%). We can test the null hypothesis that the fraction of actual violations π for a forecasting model is significantly different from α using the unconditional coverage test by Kupiec [20]. The joint null hypothesis that the VaR violations are independent and the average number of violations is correct can be tested using the conditional coverage test by Christoffersen [21]: differently from the unconditional

coverage test, the Christoffersen's test also considers the dependence of violations for consecutive days. Finally, noting that financial regulators are also concerned with the magnitude of the VaR violations, we computed the asymmetric quantile loss **(QL)** function proposed by Gonzalez–Rivera et al. [18]:

$$QL_{t+1,\alpha} = (\alpha - I_{t+1}(\alpha))(y_{t+1} - VaR_{t+1,\alpha}),$$

where $I_{t+1}(\alpha) = 1$ if $y_{t+1} < VaR_{t+1,\alpha}$ and zero otherwise.

Empirical Analysis
A. In-sample analysis
For sake of space and interest, we report in **Tables 1–4** the parameters estimates for the HAR model under different specifications – with and without the implied volatility and Google Trends, while **Table 5** summarizes the

parameters estimates across different models by showing only the estimated parameters of the implied volatility and Google Trends and their statistical significance.

In general, only the implied volatility has a significant effect on the realized volatility across most stocks and estimated models. Instead, Google Trends does not seem to have any appreciable effect on the realized volatility, thus confirming similar evidence reported by Basistha et al. [10].

B. Out-of-sample analysis
Variance forecasts. The models included in the Model Confidence Set **(MCS)** at the 10 % confidence level and their associated MSE loss are reported in **Table 6**.

The models including the implied volatility tend to have smaller MSE compared to other models, but these

				Table 1
Summary of HAR models for SBERBANK [Обзор моделей HAR для акций ПАО «Сбербанк»]				
Dependent variable: RV$_{t+1}$				
RV$_t$	$-6.16 \cdot 10^{-2}$	$-6.57 \cdot 10^{-2}$	$-6.54 \cdot 10^{-2}$	$-6.10 \cdot 10^{-2}$
	$(4.43 \cdot 10^{-2})$	$(4.40 \cdot 10^{-2})$	$(4.40 \cdot 10^{-2})$	$(4.43 \cdot 10^{-2})$
RV$_{weekly}$	$5.13 \cdot 10^{-3}***$	$5.80 \cdot 10^{-3}***$	$5.86 \cdot 10^{-3}***$	$5.26 \cdot 10^{-3}***$
	$(1.53 \cdot 10^{-3})$	$(1.53 \cdot 10^{-3})$	$(1.54 \cdot 10^{-3})$	$(1.54 \cdot 10^{-3})$
RV$_{monthly}$	$-2.52 \cdot 10^{-3}$	$-9.25 \cdot 10^{-3}**$	$-9.45 \cdot 10^{-3}**$	$-3.12 \cdot 10^{-3}$
	$(4.12 \cdot 10^{-3})$	$(4.66 \cdot 10^{-3})$	$(4.71 \cdot 10^{-3})$	$(4.22 \cdot 10^{-3})$
IV$_t$		$2.08 \cdot 10^{-7}***$	$2.05 \cdot 10^{-7}***$	
		$(6.92 \cdot 10^{-8})$	$(6.98 \cdot 10^{-8})$	
GT$_t$			$-3.26 \cdot 10^{-9}$	$-6.63 \cdot 10^{-9}$
			$(9.79 \cdot 10^{-9})$	$(9.80 \cdot 10^{-9})$
Constant	$3.10 \cdot 10^{-6}***$	$-2.19 \cdot 10^{-6}$	$-1.91 \cdot 10^{-6}$	$3.52 \cdot 10^{-6}***$
	$(8.783 \cdot 10^{-7})$	$(1.96 \cdot 10^{-6})$	$(2.14 \cdot 10^{-6})$	$(1.08 \cdot 10^{-6})$
Note: *$p < 0.1$; **$p < 0.05$; ***$p < 0.01$.				

				Table 2
Summary of HAR models for GAZPROM [Обзор моделей HAR для акций ПАО «Газпром»]				
Dependent variable: RV$_{t+1}$				
RV$_t$	$1.53 \cdot 10^{-2}$	$1.44 \cdot 10^{-2}$	$1.42 \cdot 10^{-2}$	$1.29 \cdot 10^{-2}$
	$(4.34 \cdot 10^{-2})$	$(4.35 \cdot 10^{-2})$	$(4.34 \cdot 10^{-2})$	$(4.34 \cdot 10^{-2})$
RV$_{weekly}$	$3.95 \cdot 10^{-3}$	$3.79 \cdot 10^{-3}$	$5.50 \cdot 10^{-3}$	$5.40 \cdot 10^{-3}$
	$(7.59 \cdot 10^{-3})$	$(7.60 \cdot 10^{-3})$	$(7.65 \cdot 10^{-3})$	$(7.65 \cdot 10^{-3})$
RV$_{monthly}$	$1.35 \cdot 10^{-2}$	$1.73 \cdot 10^{-2}$	$1.25 \cdot 10^{-2}$	$1.75 \cdot 10^{-2}$
	$(1.37 \cdot 10^{-2})$	$(1.49 \cdot 10^{-2})$	$(1.38 \cdot 10^{-2})$	$(1.49 \cdot 10^{-2})$
IV$_t$		$-8.08 \cdot 10^{-8}$		$-1.07 \cdot 10^{-7}$
		$(1.23 \cdot 10^{-7})$		$(1.24 \cdot 10^{-7})$
GT$_t$			$-6.33 \cdot 10^{-8}*$	$-6.78 \cdot 10^{-8}*$
			$(4.05 \cdot 10^{-8})$	$(4.08 \cdot 10^{-8})$
Constant	$2.83 \cdot 10^{-6}$	$4.44 \cdot 10^{-6}$	$5.30 \cdot 10^{-6}*$	$7.62 \cdot 10^{-6}*$
	$(2.12 \cdot 10^{-6})$	$(3.25 \cdot 10^{-6})$	$(2.64 \cdot 10^{-6})$	$(3.76 \cdot 10^{-6})$
Note: *$p < 0.1$; **$p < 0.05$; ***$p < 0.01$.				

Table 3

Summary of HAR models for YANDEX

[Обзор моделей HAR для акций компании ОО «Яндекс Н.В.»]

	Dependent variable: RV_{t+1}			
RV_t	$1.97 \cdot 10^{-2}$	$1.63 \cdot 10^{-2}$	$1.97 \cdot 10^{-2}$	$1.63 \cdot 10^{-2}$
	$(4.33 \cdot 10^{-2})$	$(4.33 \cdot 10^{-2})$	$(4.34 \cdot 10^{-2})$	$(4.34 \cdot 10^{-2})$
RV_{weekly}	$-8.02 \cdot 10^{-4}$	$-1.12 \cdot 10^{-3}$	$-8.72 \cdot 10^{-4}$	$-1.19 \cdot 10^{-3}$
	$(1.23 \cdot 10^{-3})$	$(1.24 \cdot 10^{-3})$	$(1.28 \cdot 10^{-3})$	$(1.30 \cdot 10^{-3})$
$RV_{monthly}$	$8.89 \cdot 10^{-3}$***	$8.72 \cdot 10^{-3}$***	$8.94 \cdot 10^{-3}$***	$8.78 \cdot 10^{-3}$***
	$(2.37 \cdot 10^{-3})$	$(2.37 \cdot 10^{-3})$	$(2.39 \cdot 10^{-3})$	$(2.39 \cdot 10^{-3})$
IV_t		$9.49 \cdot 10^{-8}$*		$9.49 \cdot 10^{-8}$*
		$(5.73 \cdot 10^{-8})$		$(5.73 \cdot 10^{-8})$
GT_t			$5.79 \cdot 10^{-9}$	$5.75 \cdot 10^{-9}$
			$(2.96 \cdot 10^{-8})$	$(2.96 \cdot 10^{-8})$
Constant	$-8.00 \cdot 10^{-7}$	$-4.30 \cdot 10^{-6}$*	$-8.47 \cdot 10^{-7}$	$-4.34 \cdot 10^{-6}$*
	$(1.20 \cdot 10^{-6})$	$(2.43 \cdot 10^{-6})$	$(1.22 \cdot 10^{-6})$	$(2.44 \cdot 10^{-6})$

Note: *$p < 0.1$; **$p < 0.05$; ***$p < 0.01$.

Table 4

Summary of HAR models for ROSNEFT

[Обзор моделей HAR для акций ПАО «Роснефть»]

	Dependent variable: RV_{t+1}			
RV_t	$1.09 \cdot 10^{-2}$	$9.03 \cdot 10^{-3}$	$1.06 \cdot 10^{-2}$	$8.89 \cdot 10^{-3}$
	$(4.36 \cdot 10^{-2})$	$(4.37 \cdot 10^{-2})$	$(4.37 \cdot 10^{-2})$	$(4.37 \cdot 10^{-2})$
RV_{weekly}	$2.01 \cdot 10^{-2}$***	$2.00 \cdot 10^{-2}$***	$1.97 \cdot 10^{-2}$***	$1.98 \cdot 10^{-2}$***
	$(4.00 \cdot 10^{-3})$	$(4.00 \cdot 10^{-3})$	$(4.06 \cdot 10^{-3})$	$(4.06 \cdot 10^{-3})$
$RV_{monthly}$	$-9.83 \cdot 10^{-3}$	$-6.52 \cdot 10^{-3}$	$-9.22 \cdot 10^{-3}$	$-6.24 \cdot 10^{-3}$
	$(7.58 \cdot 10^{-3})$	$(8.23 \cdot 10^{-3})$	$(7.70 \cdot 10^{-3})$	$(8.29 \cdot 10^{-3})$
IV_t		$-9.82 \cdot 10^{-8}$		$-9.37 \cdot 10^{-8}$
		$(9.55 \cdot 10^{-8})$		$(9.66 \cdot 10^{-8})$
GT_t			$1.19 \cdot 10^{-8}$	$8.15e{-}09$
			$(2.56 \cdot 10^{-8})$	$(2.59 \cdot 10^{-8})$
Constant	$2.31 \cdot 10^{-6}$	$4.51 \cdot 10^{-6}$	$1.93 \cdot 10^{-6}$	$4.15 \cdot 10^{-6}$
	$(1.685 \cdot 10^{-6})$	$(2.725 \cdot 10^{-6})$	$(1.877 \cdot 10^{-6})$	$(2.960 \cdot 10^{-6})$

Note:$p < 0.1$; **$p < 0.05$; ***$p < 0.01$.

Table 5

Summary of the estimated parameters of the implied volatility and Google Trends across different models

[Оценочные параметры вмененной волатильности и Google Trends для разных моделей]

SBERBANK	HAR	ARFIMA	GARCH	GAZPROM	HAR	ARFIMA	GARCH
IV	$2.05 \cdot 10^{-7}$***	$1.50 \cdot 10^{-5}$	$6.23 \cdot 10^{-7}$	IV	$-1.07 \cdot 10^{-7}$	$3.00 \cdot 10^{-6}$**	$7.15 \cdot 10^{-9}$
GT	$-3.26 \cdot 10^{-9}$	$1.00 \cdot 10^{-5}$	$2.12 \cdot 10^{-7}$	GT	$-6.78 \cdot 10^{-8}$*	$-1.00 \cdot 10^{-6}$	$3.28 \cdot 10^{-9}$
YANDEX	HAR	ARFIMA	GARCH	ROSNEFT	HAR	ARFIMA	GARCH
IV	$9.49 \cdot 10^{-8}$*	$4.01 \cdot 10^{-6}$***	$6.62 \cdot 10^{-7}$	IV	$-9.37 \cdot 10^{-8}$	$1.60 \cdot 10^{-6}$	$5.67 \cdot 10^{-8}$
GT	$5.75 \cdot 10^{-9}$	$2.00 \cdot 10^{-8}$	$5.34 \cdot 10^{-8}$	GT	$8.15 \cdot 10^{-9}$	$1.20 \cdot 10^{-6}$	$4.98 \cdot 10^{-8}$

Note: *$p < 0.1$; **$p < 0.05$; ***$p < 0.01$.

Table 6

Models included in the MCS at the 10 % confidence level and associated mean squared loss
[Модели, попадающие в 10% доверительный интервал и соответствующие среднеквадратичные отклонения]

Models in the MCS	Rank	Loss	Models in the MCS	Rank	Loss
SBERBANK			**GAZPROM**		
HAR	8	$2.18 \cdot 10^{-10}$	HAR	1	$5.87 \cdot 10^{-11}$
HAR + IV	6	$1.86 \cdot 10^{-10}$	HAR + IV	2	$5.87 \cdot 10^{-11}$
HAR + GT	9	$2.19 \cdot 10^{-10}$	HAR + GT	5	$6.09 \cdot 10^{-11}$
HAR + IV + GT	7	$1.86 \cdot 10^{-10}$	HAR + IV + GT	4	$6.08 \cdot 10^{-11}$
ARFIMA	5	$1.65 \cdot 10^{-10}$	GARCH	3	$6.06 \cdot 10^{-11}$
ARFIMA + IV	4	$1.65 \cdot 10^{-10}$	GARCH + IV	6	$6.12 \cdot 10^{-11}$
GARCH	2	$1.62 \cdot 10^{-10}$	*Number of models eliminated: 6*		
GARCH + IV	1	$1.61 \cdot 10^{-10}$			
GARCH + GT	3	$1.65 \cdot 10^{-10}$			
Number of models eliminated: 3					
YANDEX			**ROSNEFT**		
HAR	1	$5.24 \cdot 10^{-11}$	HAR	8	$6.57 \cdot 10^{-11}$
HAR + IV	2	$5.26 \cdot 10^{-11}$	HAR + IV	6	$6.51 \cdot 10^{-11}$
HAR + GT	4	$5.28 \cdot 10^{-11}$	HAR + GT	7	$6.55 \cdot 10^{-11}$
HAR + IV + GT	6	$5.31 \cdot 10^{-11}$	HAR + IV + GT	5	$6.50 \cdot 10^{-11}$
ARFIMA	3	$5.27 \cdot 10^{-11}$	ARFIMA	3	$6.14 \cdot 10^{-11}$
ARFIMA + IV	5	$5.28 \cdot 10^{-11}$	ARFIMA + GT	9	$6.79 \cdot 10^{-11}$
GARCH	7	$5.34 \cdot 10^{-11}$	GARCH	1	$5.85 \cdot 10^{-11}$
Number of models eliminated: 5			GARCH + GT	4	$6.47 \cdot 10^{-11}$
			GARCH + GT + IV	2	$5.79 \cdot 10^{-11}$
			Number of models eliminated: 3		

Table 7

Kupiec tests p-values and Christoffersen's tests p-values
[P-величины в тестах Kupiec и Christoffersen]

	VaR with $\alpha = 5$ %		VaR with $\alpha = 1$ %			VaR with $\alpha = 5\%$		VaR with $\alpha = 1\%$	
	Kupiec t.	Christ. t.	Kupiec t.	Christ. t.		Kupiec t.	Christ. t.	Kupiec t.	Christ. t.
SBERBANK					**GAZPROM**				
HAR	**0.04**	0.10	0.19	0.23	HAR	**0.00**	**0.00**	0.71	0.07
HAR + IV	**0.04**	0.10	0.19	0.23	HAR + IV	**0.00**	**0.00**	0.71	0.07
HAR + GT	0.32	0.25	**0.04**	0.10	HAR + GT	0.48	0.25	**0.04**	0.11
HAR + IV + GT	0.32	0.25	**0.04**	0.10	HAR + IV + GT	0.72	0.20	**0.01**	**0.03**
ARFIMA	**0.03**	0.07	0.23	0.42	ARFIMA	0.72	0.20	**0.01**	**0.00**
ARFIMA + IV	0.07	0.09	0.23	0.42	ARFIMA + IV	0.00	0.00	0.00	0.00
ARFIMA + GT	0.05	0.12	0.23	0.42	ARFIMA + GT	0.01	0.00	0.01	0.00
ARFIMA + IV + GT	0.05	0.09	0.23	0.42	ARFIMA + IV + GT	0.72	0.20	**0.01**	**0.00**
GARCH	**0.03**	0.07	**0.04**	0.10	GARCH	0.01	0.00	0.01	0.03
GARCH + IV	**0.01**	**0.04**	**0.04**	0.10	GARCH + IV	0.01	0.00	0.01	0.03
GARCH + GT	0.05	0.12	**0.04**	0.10	GARCH + GT	0.05	**0.04**	**0.01**	**0.03**
GARCH + IV + GT	**0.01**	0.12	**0.04**	0.10	GARCH + IV + GT	0.01	0.03	0.01	0.03
YANDEX					**ROSNEFT**				
HAR	**0.04**	0.11	0.99	0.66	HAR	**0.04**	0.11	**0.00**	**0.00**
HAR + IV	**0.04**	0.11	0.99	0.66	HAR + IV	0.01	0.03	0.00	0.00
HAR + GT	0.99	0.66	**0.01**	**0.03**	HAR + GT	0.48	0.70	0.76	0.93
HAR + IV + GT	0.99	0.66	**0.01**	**0.03**	HAR + IV + GT	0.48	0.70	0.76	0.93
ARFIMA	0.44	0.59	0.81	0.70	ARFIMA	0.48	0.40	0.76	0.93
ARFIMA + IV	0.31	0.56	0.72	0.64	ARFIMA + IV	0.00	0.00	0.00	0.00
ARFIMA + GT	0.44	0.59	**0.02**	**0.03**	ARFIMA + GT	0.18	0.40	0.32	0.58
ARFIMA + IV + GT	0.44	0.59	**0.02**	**0.03**	ARFIMA + IV + GT	0.48	0.40	0.76	0.93
GARCH	0.99	0.66	**0.01**	**0.03**	GARCH	0.30	0.26	0.61	0.87
GARCH + IV	0.44	0.59	**0.01**	**0.03**	GARCH + IV	0.72	0.55	0.07	0.20
GARCH + GT	0.44	0.59	**0.01**	**0.03**	GARCH + GT	0.24	0.43	0.61	0.87
GARCH + IV + GT	0.44	0.59	**0.01**	**0.03**	GARCH + IV + GT	0.72	0.55	0.07	0.20

Note: P-values smaller than 0.05 are in bold font.

differences are not statistically significant and several competing models are also included into the MCS. Interestingly, the simple HAR and GARCH models without additional regressors have the smallest MSE for 3 stocks out of 4, thus showing that efficiency gains more than compensate any possible model misspecifications and parameters biases.

Value-at-Risk forecasts. The p-values of the Kupiec and Christoffersen's tests are reported in **Table 7**, while the models included in the Model Confidence Set **(MCS)** at the 10% confidence level and their associated asymmetric quantile loss are reported in **Table 8**.

These tables show that ARFIMA and HAR models without additional regressors tend to be the best compromise for precise VaR forecasts, particularly at the 1% level, which is the most important quantile for regulatory purposes. The HAR model with the implied volatility showed in several cases the lowest asymmetric quantile losses, thus confirming the previous in-sample analysis. However, the differences with the other models were rather small and not statistically significant. Moreover, for two stocks (Yandex and Rosneft) the models with the implied volatility were excluded from the

MCS for the VaR at the 1 % probability level. These results again highlight that simpler models are a better choice when out-of-sample forecasting is the main concern, thanks to more efficient estimates in comparison to more complex specifications.

Conclusions

Three volatility forecasting models and several different specifications, including also the implied volatility computed from option prices and Google Trends data, were used to model and forecast the realized volatility and the VaR of four Russian stocks.

The in-sample analysis showed that only the implied volatility had a significant effect on the realized volatility across most stocks and estimated models, whereas Google Trends did not have any significant effect on the realized volatility. The out-of-sample analysis highlighted that the models including the implied volatility had smaller MSE compared to several competing models, but these differences were not statistically significant. Moreover, the simple HAR and GARCH models without additional regressors showed the smallest MSE for three stocks out of four, thus showing that efficiency gains more than compensate any possible model

Table 8

Models included in the MCS at the 10 % confidence level and associated asymmetric quantile loss
[Модели, попадающие в 10% доверительный интервал и соответствующие асимметричная отклонения квантиля]

VaR with α =5%			VaR with α =1%			VaR with α =5%			VaR with α =1%		
Models in MCS	Rank	Loss	Models in MCS	Rank	Loss	Models in MCS	Rank	Loss	Models in MCS	Rank	Loss
SBERBANK						**GAZPROM**					
HAR	4	$2.68 \cdot 10^{-4}$	HAR	4	$2.68 \cdot 10^{-4}$	HAR	7	$3.24 \cdot 10^{-4}$	HAR	1	$2.69 \cdot 10^{-4}$
HAR + IV	1	$2.61 \cdot 10^{-4}$	HAR + IV	1	$2.61 \cdot 10^{-4}$	HAR + IV	8	$3.28 \cdot 10^{-4}$	HAR + IV	2	$2.69 \cdot 10^{-4}$
HAR + GT	3	$2.62 \cdot 10^{-4}$	HAR + GT	3	$2.62 \cdot 10^{-4}$	HAR + GT	4	$2.82 \cdot 10^{-4}$	GARCH	3	$2.91 \cdot 10^{-4}$
HAR + IV + GT	2	$2.61 \cdot 10^{-4}$	HAR + IV + GT	2	$2.61 \cdot 10^{-4}$	HAR + IV + GT	3	$2.79 \cdot 10^{-4}$	*N. of models eliminated: 9*		
ARFIMA	9	$2.97 \cdot 10^{-4}$	ARFIMA	6	$2.78 \cdot 10^{-4}$	ARFIMA	1	$2.69 \cdot 10^{-4}$			
ARFIMA + IV	7	$2.84 \cdot 10^{-4}$	ARFIMA + IV	5	$2.75 \cdot 10^{-4}$	ARFIMA + IV	9	$4.05 \cdot 10^{-4}$			
GARCH	6	$2.82 \cdot 10^{-4}$	*N. of models eliminated: 6*			GARCH	6	$2.91 \cdot 10^{-4}$			
GARCH + IV	5	$2.82 \cdot 10^{-4}$				GARCH + IV	5	$2.91 \cdot 10^{-4}$			
GARCH + GT	8	$2.99 \cdot 10^{-4}$				GARCH + GT	2	$2.75 \cdot 10^{-4}$			
N. of models eliminated: 3						*N. of models eliminated: 3*					
YANDEX						**ROSNEFT**					
HAR	9	$2.50 \cdot 10^{-4}$	HAR	1	$2.31 \cdot 10^{-4}$	HAR + GT	1	$2.55 \cdot 10^{-4}$	ARFIMA	1	$6.03 \cdot 10^{-5}$
HAR + IV	1	$2.20 \cdot 10^{-4}$	*N. of m. eliminated: 11*			HAR + IV + GT	2	$2.56 \cdot 10^{-4}$	ARFIMA + GT	3	$6.78 \cdot 10^{-5}$
HAR + GT	3	$2.23 \cdot 10^{-4}$				ARFIMA	5	$2.61 \cdot 10^{-4}$	GARCH	2	$6.31 \cdot 10^{-5}$
HAR + IV + GT	6	$2.26 \cdot 10^{-4}$				ARFIMA + GT	7	$2.69 \cdot 10^{-4}$	*N. of models eliminated: 9*		
ARFIMA	5	$2.24 \cdot 10^{-4}$				GARCH	3	$2.59 \cdot 10^{-4}$			
ARFIMA + IV	7	$2.26 \cdot 10^{-4}$				GARCH + GT	6	$2.68 \cdot 10^{-4}$			
GARCH	4	$2.24 \cdot 10^{-4}$				GARCH + IV + GT	4	$2.59 \cdot 10^{-4}$			
GARCH + IV	2	$2.21 \cdot 10^{-4}$				*N. of models eliminated: 5*					
GARCH + GT	8	$2.27 \cdot 10^{-4}$									
N. of models eliminated: 3											

misspecifications and parameters biases. A similar result was also found after performing a backtesting analysis with daily VaR forecasts, where ARFIMA and HAR models without additional regressors had the best results in several cases (particularly at the 1 % probability level), whereas the HAR model with implied volatility displayed good results when forecasting the VaR at the 5 % probability level. Therefore, these findings revealed that Google Trends data did not improve the forecasting performances of the models when a market-based predictor like the implied volatility was included, thus confirming similar results reported by Basistha et al [10].

How to explain these results? One possible explanation was proposed by [10], who put forward the idea that the informational content included in the internet search activity is also present in the implied volatility, but the opposite is not true. This should not come as a surprise, if we consider that implied volatility is a forward-looking measure mainly based on the expectations of institutional investors and market makers who have access to premium and insider information, while Google Trends data are mainly based on the expectations of small investors and un-informed traders. A second simpler explanation is that Yandex is the main search engine in Russia with a market share close to 56 % in 2018 (all platforms), while Google is second with a market share close to 42 %, so that Google Trends may not be the best proxy for Russian investors' interest and behavior. More research is definitely needed in this regard, and this issue is left as an avenue of future research.

References

1. Bauwens L., Hafner C.M., Laurent S. Handbook of volatility models and their applications. Wiley, 2012. 548 p.

2. Campos I., Cortazar G., Reyes T. Modeling and predicting oil VIX: Internet search volume versus traditional variables. *Energy Economics*. 2017. No. 66. Pp. 194–204. DOI: 10.1016/j.eneco.2017.06.009

3. Donaldson R.G., Kamstra M.J. Volatility forecasts, trading volume, and the arch versus option-implied volatility trade-off. *Journal of Financial Research*. 2005. Vol. 28. No. 4. Pp. 519–538.

4. Andrei D., Hasler M. Investor attention and stock market volatility. *The Review of Financial Studies*. 2014. Vol. 28. No. 1. Pp. 33–72.

5. Vlastakis N. Markellos R.N. Information demand and stock market volatility. *Journal of Banking and Finance*. 2012. Vol. 36. No. 6. Pp. 1808–1821.

6. Mayhew S. Implied volatility. *Financial Analysts Journal*. 1995. Vol. 51. No. 4. Pp. 8–20.

7. Corsi F. A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics*. 2009. Vol. 7. No. 2. Pp. 174–196 .

8. Andersen T.G., Bollerslev T., Diebold F.X., Labys P. Modeling and forecasting realized volatility. *Econometrica*. 2003. Vol. 71. No. 2. Pp. 579–625.

9. Goddard J., Kita A., Wang Q. Investor attention and FX market volatility. *Journal of International Financial Markets, Institutions and Money*. 2015. No. 38. Pp. 79–96.

10. Basistha A., Kurov A., Wolfe M. Volatility Forecasting: The Role of Internet Search Activity and Implied Volatility. West Virginia University working paper. Avalable at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2812387 (accessed: 05.02.2019).

11. Barndorff-Nielsen O.E., Shephard N. Econometric analysis of realized volatility and its use in estimating stochastic volatility models. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*. 2002. Vol. 64. No. 2. Pp. 253–280.

12. Liu L.Y., Patton A.J., Sheppard K. Does anything beat 5-minute RV? A comparison of realized measures across multiple asset classes. *Journal of Econometrics*. 2015. Vol. 187. No. 1. Pp. 293–311.

13. Hull J.C. Options, futures and other derivatives. Pearson, 2018. 894 p.

14. Fengler M.R. Semiparametric modeling of implied volatility. Berlin: Springer, 2005. 231 p.

15. Hyndman R.J., Khandakar Y. Automatic Time Series Forecasting: the forecast Package for R. *Journal of Statistical Software*. 2008. Vol. 27. No. 3. DOI: 10.18637/jss.v027.i03

16. Hansen P.R., Lunde A. A forecast comparison of volatility models: does anything beat a GARCH (1,1)? *Journal of Applied Econometrics*. 2005. Vol. 20. No. 7. Pp. 873–889.

17. Hansen P.R., Lunde A., Nason J.M. The model confidence set. *Econometrica*. 2011. Vol. 79. No. 2. Pp. 453–497.

18. González-Rivera G., Lee T.H., Mishra S. Forecasting volatility: A reality check based on option pricing, utility function, value-at-risk, and predictive likelihood. *International Journal of Forecasting*. 2004. Vol. 20. No. 4. Pp. 629–645.

19. Louzis D.P., Xanthopoulos-Sisinis S., Refenes A.P. Realized volatility models and alternative Value-at-Risk prediction strategies. *Economic Modelling*. 2014. No. 40. Pp. 101–116.

20. Kupiec P.H. Techniques for Verifying the Accuracy of Risk Measurement Models. *The Journal of Derivatives*. 1995. Vol. 3. No. 2. Pp. 73–84.

21. Christoffersen P.F. Evaluating interval forecasts. *International economic review*. 1998. No. 39. Pp. 841–862.

Прогнозирование реализованной волатильности котируемых российских акций с помощью инструмента Google Trends и вмененной волатильности

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Рассмотрено прогнозирование реализованной волатильности (Realized Volatility, RV) и стоимости под риском (Value-at-Risk, VaR) наиболее ликвидных российских акций с помощью моделей GARCH, ARFIMA и HAR, используя вмененную волатильность (implied volatility), рассчитанную исходя из цен опционов, а также данные Google Trends. Анализ в пределах выборки показывает, что только вмененная волатильность оказывает существенное влияние на реализованную волатильность большинства акций, в то время как данные Google Trends не оказывают существенного влияния. Анализ за пределами выборки выявил, что модели, основанные на вмененной волатильности, ещё лучше прогнозируют реализованную волатильность, тогда как модели, построенные на активности интернет-запросов, в некоторых случая прогнозируют ещё хуже. Более того, простые модели HAR и ARFIMA без дополнительных регрессоров зачастую лучше прогнозируют дневную реализованную волатильность и дневную стоимость под риском на уровне 1 %, таким образом демонстрируя, что эффективность модели компенсирует возможные ошибки в спецификации модели и смещение параметров. Наши расчеты показывают, что, в случае, российских котируемых акций, данные Google Trends не несут дополнительной информации, не учтенной уже во вмененной волатильности.

Ключевые слова: прогнозирование, реализованная волатильность, стоимость под риском, вмененная волатильность, Google Trends, GARCH, ARFIMA, HAR

Библиографический список

1. *Bauwens L., Hafner C.M., Laurent S.* Handbook of volatility models and their applications. Wiley, 2012. 548 p.

2. *Campos I., Cortazar G., Reyes T.* Modeling and predicting oil VIX: Internet search volume versus traditional variables // Energy Economics. 2017. N 66. P. 194–204. DOI: 10.1016/j.eneco.2017.06.009

3. *Donaldson R.G., Kamstra M.J.* Volatility forecasts, trading volume, and the arch versus option-implied volatility trade-off // Journal of Financial Research. 2005. V. 28. N 4. P. 519–538.

4. *Andrei D., Hasler M.* Investor attention and stock market volatility // The Review of Financial Studies. 2014. V. 28. N 1. P. 33–72.

5. *Vlastakis N. Markellos R.N.* Information demand and stock market volatility // Journal of Banking and Finance. 2012. V. 36. N 6. P. 1808–1821.

6. *Mayhew S.* Implied volatility // Financial Analysts Journal. 1995. V. 51. N 4. P. 8–20.

7. *Corsi F.* A simple approximate long-memory model of realized volatility // Journal of Financial Econometrics. 2009. V. 7. N 2. P. 174–196.

8. *Andersen T.G., Bollerslev T., Diebold F.X., Labys P.* Modeling and forecasting realized volatility // Econometrica. 2003. V. 71. N 2. P. 579–625.

9. *Goddard J., Kita A., Wang Q.* Investor attention and FX market volatility // Journal of International Financial Markets, Institutions and Money. 2015. N 38. P. 79–96.

10. *Basistha A., Kurov A., Wolfe M.* Volatility Forecasting: The Role of Internet Search Activity and Implied Volatility. West Virginia University working paper. Avalable at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2812387 (accessed: 05.02.2019).

11. *Barndorff-Nielsen O.E., Shephard N.* Econometric analysis of realized volatility and its use in estimating stochastic volatility models // Journal of the Royal Statistical Society: Series B (Statistical Methodology). 2002. V. 64. N 2. P. 253–280.

12. *Liu L.Y., Patton A.J., Sheppard K.* Does anything beat 5-minute RV? A comparison of realized measures across multiple asset classes // Journal of Econometrics. 2015. V. 187. N 1. P. 293–311.

13. *Hull J.C.* Options, futures and other derivatives. Pearson, 2018. 894 p.

14. *Fengler M.R.* Semiparametric modeling of implied volatility. Berlin: Springer, 2005. 231 p.

15. *Hyndman R.J., Khandakar Y.* Automatic Time Series Forecasting: the forecast Package for R. // Journal of Statistical Software. 2008. V. 27. N 3. DOI: 10.18637/jss.v027.i03

16. *Hansen P.R., Lunde A.* A forecast comparison of volatility models: does anything beat a GARCH (1,1)? // Journal of Applied Econometrics. 2005. V. 20. N 7. P. 873–889.

17. *Hansen P.R., Lunde A., Nason J.M.* The model confidence set // Econometrica. 2011. V. 79. N 2. P. 453–497.

18. *González-Rivera G., Lee T.H., Mishra S.* Forecasting volatility: A reality check based on option pricing, utility function, value-at-risk, and predictive likelihood // International Journal of Forecasting. 2004. V. 20. N 4. P. 629–645.

19. *Louzis D.P., Xanthopoulos-Sisinis S., Refenes A.P.* Realized volatility models and alternative Value-at-Risk prediction strategies // Economic Modelling. 2014. N 40. P. 101–116.

20. *Kupiec P.H.* Techniques for Verifying the Accuracy of Risk Measurement Models // The Journal of Derivatives. 1995. V. 3. N 2. P. 73–84.

21. *Christoffersen P.F.* Evaluating interval forecasts // International economic review. 1998. N 39. P. 841–862.