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Monopolies and perfect competition in Solow–Uzawa's general equilibrium growth model

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Abstract. The purpose of this study is to introduce monopolies to neoclassical growth theory. This unique contribution attempts to make neoclassical economic growth theory more realistic in modelling the complexity of economic growth and development with different types of market structures. This study is based on a few well-established economic theories in the literature of economics. We frame the model on basis of the Solow–Uzawa two-sector growth model. The modelling of monopoly is based on well-developed monopoly theory. We model behavior of the household with Zhang's concept of disposable income and utility function. The model endogenously determines profits of monopolies which are equally distributed among the homogeneous population. We build the model and then identify the existence of an equilibrium point by simulation. We conduct comparative static analyses in some parameters.

Keywords: monopoly, perfect competition, capital accumulation, Solow model, Uzawa model, profit

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Монополии и совершенная конкуренция в общем уравнении модели роста Солоу–Узавы

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Аннотация. В статье исследовано интегрирование монополий в неоклассическую теорию роста. Автор пытается сделать неоклассическую теорию экономического роста более реалистичной, сопоставив сложности экономического роста и развития с различными типами рыночных структур. Исследование основано на нескольких, хорошо зарекомендовавших себя в литературе, экономических теориях. Создана модель на основе двухсекторной модели роста Солоу–Узавы. Моделирование монополии основано на теории монополии, хорошо обоснованной в экономической литературе. Показано наиболее вероятное поведение домохозяйства с помощью концепции располагаемого дохода и функции полезности Чжана. Модель определяет прибыль монополий на внутреннем рынке, которая равномерно распределяется среди населения. Смоделировано существование точки равновесия, проведен сравнительный статический анализ по некоторым параметрам.

Ключевые слова: монополия, совершенная конкуренция, накопление капитала, модель Солоу, модель Узавы, прибыль

1. Introduction

Different market structures, such as perfect competition, imperfect competition, oligopoly, and monopoly co-exist in contemporary economies. Different economic theories analyze efficiencies and equilibrium of different market structures under varied economic institutions. Microeconomics has made great contributions to functionings of different markets in modern times (e.g., Nikaido, 1975 [1]; Mas-Colell, *et al.* 1995 [2]; Brakman and Heijdra, 2004 [3]; Wang, 2012 [4]; Behrens and Murata, 2007 [5], 2009 [6]; and Parenti, *et.al*. 2017 [7]). Nevertheless, there is a lack of integration of these microeconomic theories with macroeconomics. The purpose of this study is to a unique contribution to economic growth theory by introducing monopolies to neoclassical growth theory. It attempts to make neoclassical economic growth theory more realistic in modelling the complexity of economic growth and development with different types of market structures. This study is based on a few well-established economic theories in the literature of economics. We frame the model on basis of the Solow–Uzawa two-sector growth model. The modelling of monopoly is based on well-developed monopoly theory.

Neoclassical growth theory is a main modelling framework of economic growth and development with microeconomic foundation. A main character of the theory is that it treats endogenous physical capital as the machine of economic growth. Nevertheless, most of formal models in the literature of economic theory are developed for economies with perfectly competitive markets. Neoclassical growth theory is a well-developed economic theory with endogenous wealth and physical capital built on microeconomic foundation. As discussed extensively in Zhang (2005) [8], neoclassical economic theory fails to be integrated with different microeconomic theories partly due to analytical difficulties in association of integrating. The approach makes it analytically difficult to analyze behavior of households. Zhang (1993 [9], 2005 [8]) applies an alternative approach to modelling household behavior. This approach has been applied to different economic problems. This study is another application of the approach to deal with a complicated issue in economic theory – how to take account of different market structures in neoclassical growth theory. We are concerned with two core models in economic theory as the basic framework. They are respectively neoclassical growth theory with perfect competition (Solow, 1956 [10]; Uzawa, 1961 [11]) and theory of monopoly. The two models have resulted in two extensive but separate literatures. The literature of neoclassical economics of perfect competition initiated with the Solow–Uzawa model is extensive (e.g., Burmeister and Dobell 1970 [12]; Azariadis, 1993 [13]; Barro and Sala-i-Martin, 1995 [14]; Jensen and Larsen, 2005 [15]; Ben-David and Loewy, 2003 [16], and Zhang, 2005 [8], 2008 [17]). Economists have made great efforts in integrating microeconomic theories and macroeconomics. New economic theory is a main attempt in introducing imperfect competition to macroeconomics (e.g., Dixit and Stiglitz, 1977 [18]; Krugman, 1979 [19]; Romer, 1990 [20]; Benassy, 1996 [21]; Wang, 2012 [4]; Nocco, *et. al.*, 2017 [22]). New economic growth includes perfect as well imperfect competition. But a main issue with new economic growth is that it lacks a proper mechanism of physical capital and wealth accumulation. Zhang (2018 [23]) attempts to integrate new growth theory and neoclassical growth theory. Nevertheless, these studies in new growth theory don't integrate monopoly theory to formal growth theory. This study introduces monopoly theory to neoclassical growth theory with capital accumulation. The rest of the paper is organized as follows. Section 2 builds a growth model of endogenous capital accumulation with perfect competition and monopoly. Section 3 studies analytical properties of the model and identifies the existence of an equilibrium point. Section 4 carries out comparative static analysis in a few parameters. Section 5 concludes the study.

2. The growth model with monopoly

The basic contribution of this study is to introduce monopoly into the Solow–Uzawa neoclassical growth model and monopoly theory with Zhang's concept of disposable income and utility function. Most of the model are basically following the Solow–Uzawa neoclassical growth model, except modelling behavior of the household and behavior of monopoly. In our economy there are three goods and services – final goods and two monopoly products. The final goods sector produces capital goods, which is the same as in the Solow model and can be invested and consumed. The final goods sector is following the Solow model in which all markets are perfectly competitive. We follow the Uzawa two-sector modelling structure to add monopolies to neoclassical growth theory. In our model all input factors are competitive. There are two monopolies, each of them producing a single homogenous product. Monopoly product is solely consumed by consumers. Capital is used as inputs in producing final

goods and monopoly products. Labor is distributed between production of final goods and monopoly products. We consider that the economy has only two monopolies, which produce different products. It can be seen that it is straightforward to deal with cases of many monopolies. In perfect markets (homogenous) firms have zero profit, while monopolies might have positive profits. For simplicity of analysis, profits are equally shared among the homogenous households. There is no free entry in monopoly products. The final good is chosen to serve as a medium of exchange and is taken as numeraire. We assume that capital depreciates at a constant exponential rate δ_k.

The production of final product

We use $F_i(t)$, $K_i(t)$ and $N_i(t)$ to represent, respectively, output of the final goods sector, capital input and labor input. The production function of final goods is as follows:

$$F_i(t)=A_i K_i^{\alpha_i}(t)N_i^{\beta_i}(t), 0<\alpha_i,\beta_i,\alpha_i+\beta_i=1, \quad (1)$$

where A_i, α_i and β_i are parameters. We denote $w(t)$ and $r(t)$ the wage rate and the rate of interest. The profit of the final goods sector is:

$$\pi_i(t)=F_i(t)-\left(r(t)+\delta_k\right)K_i(t)-w(t)N_i(t).$$

The marginal conditions imply:

$$r_\delta(t)=\frac{\alpha_i F_i(t)}{K_i(t)}, w(t)=\frac{\beta_i F_i(t)}{N_i(t)}, \quad (2)$$

where $r_\delta(t)=r(t)+\delta_k$.

Consumer behaviors and wealth dynamics

This study applies the approach to modeling behavior of households proposed by Zhang (1993, 2005). Let $\bar{k}(t)$ stand for per capita wealth. We have $\bar{k}(t)=K(t)/\bar{N}$, where $K(t)$ is the total capital. We assume that the profit is equally shared among households. It should be noted that in new growth theory profit is often assumed to be invested for innovation. This study assumes profit to be shared equally between the homogenous households. A more general approach should specify different possible distributions of profits among firms, households and governments (for instance, in form of taxation). Let h stand for human capital. We use $\pi_j(t)$ to stand for monopoly j's profit. The current income of the representative household is:

$$y(t)=r(t)\bar{k}(t)+hw(t)+\frac{\pi_1(t)+\pi_2(t)}{\bar{N}}. \quad (3)$$

The household disposable income $\hat{y}(t)$ is the sum of the current disposable income and the value of wealth as follows:

$$\hat{y}(t)=y(t)+\bar{k}(t)= \\ =R(t)\bar{k}(t)+hw(t)+\frac{\pi_1(t)+\pi_2(t)}{\bar{N}}, \quad (4)$$

where $R(t)\equiv 1+r(t)$.

The representative household distributes the total available budget between consumption of monopoly product $c_j(t)$, and consumption of final goods $c_i(t)$, and savings $s(t)$. The budget constraint is:

$$p_1(t)\,c_1(t)+p_2(t)\,c_2(t)+c_i(t)+s(t)=\hat{y}(t). \quad (5)$$

where $p_j(t)$ is the price of monopoly product j. We assume that utility level $U(t)$ is dependent on $c_i(t)$, $c_i(t)$, and $s(t)$ as follows:

$$U(t)=\left(\xi_1 c_1^{\xi_0}(t)+\xi_2 c_2^{\xi_0}(t)+\xi_i c_i^{\xi_0}(t)\right)^{1/\xi_0} \\ s^{\lambda_0}(t),\xi_j,\xi_i,\xi_0,\lambda_0>0, \quad (6)$$

where λ_0 is called the propensity to save. As shown in Appendix A1, we solve the optimal problem as follows:

$$s(t)=\frac{\lambda_0\tilde{P}(t)\hat{y}(t)}{\bar{P}(t)+\lambda_0\tilde{P}(t)}, c_i(t)=\frac{\hat{y}(t)}{\bar{P}(t)+\lambda_0\tilde{P}(t)}, \\ c_j(t)=\frac{P_j(t)\hat{y}(t)}{\bar{P}(t)+\lambda_0\tilde{P}(t)}, \quad (7)$$

where

$$\bar{P}(t)\equiv p_1(t)P_1(t)+p_2(t)P_2(t)+1, \tilde{P}(t)\equiv \\ \equiv\bar{\xi}_1 P_1^{\xi_0}(t)+\bar{\xi}_2 P_2^{\xi_0}(t)+1.$$

$$P_j(t)\equiv\bar{\xi}_j^{-\xi_4}\,p_j^{\xi_4}(t), \bar{\xi}_j\equiv\frac{\xi_j}{\xi_i}, \xi_4\equiv\frac{1}{\xi_0-1}.$$

We see that the behavior of the household is determined once we solve $p_j(t)$ and $\hat{y}(t)$.

Wealth accumulation

According to the definition of $s(t)$, the change in the household's wealth is given by:

$$\dot{\bar{k}}(t)=s(t)-\bar{k}(t)=\frac{\lambda_0\tilde{P}(t)\hat{y}(t)}{\bar{P}(t)+\lambda_0\tilde{P}(t)}-\bar{k}(t). \quad (8)$$

This equation implies that the change in wealth is equal to saving minus dissaving.

Equilibrium for monopoly product

We use $F_j(t)$ to stand for the output of monopoly j. The equilibrium condition for monopoly product is given by:

$$c_j(t)\bar{N}=F_j(t), j=1,2. \quad (9)$$

The behavior of the monopolies

The monopoly power implies that the price is determined by the single player, monopoly, in the market. This character makes it analytically difficult to properly analyze the role of the monopoly in economic growth theory with microeconomic foundation. We now model behavior of monopolies. We use $F_j(t)$, $K_j(t)$ and $N_j(t)$ to represent respectively the output of monopoly j, its capital input and labor input. We specify the production function of the monopoly as follows:

$$F_j(t)=A_j K_j^{\alpha_j}(t)N_j^{\beta_j}(t), 0<\alpha_j,\beta_j<1, \quad (10)$$

where A_j, α_j and β_j are parameters. The profit of monopoly j is

$$\pi_j(t)=p_j(t)F_j(t)-r_\delta(t)K_j(t)-w(t)N_j(t). \quad (11)$$

From (9) and (7) we have:

$$F_j(t)=c_j(t)\bar{N}=\frac{P_j(t)\hat{y}(t)\bar{N}}{\bar{P}(t)+\lambda_0\tilde{P}(t)}. \quad (12)$$

From (4) and (11)

$$\hat{y}(t)=\tilde{y}(t)+\frac{p_1(t)F_1(t)+p_2(t)F_2(t)}{\bar{N}}, \quad (13)$$

where

$$\tilde{y}(t)=R(t)\bar{k}(t)+hw(t)-\frac{r_\delta(t)\left(K_1(t)+K_2(t)\right)+w(t)\left(N_1(t)+N_2(t)\right)}{\bar{N}}.$$

Equations (12) and (13) implies:

$$F_j(t)=\frac{P_j(t)\left(\tilde{y}(t)\bar{N}+p_1(t)F_1(t)+p_2(t)F_2(t)\right)}{\bar{P}(t)+\lambda_0\tilde{P}(t)}. \quad (14)$$

From (14) we have:

$$\frac{F_1(t)}{F_2(t)}=\frac{P_1(t)}{P_2(t)}.$$

That is

$$\frac{p_1(t)}{p_2(t)}=f\left(F_1(t),F_2(t)\right)\equiv\xi\left(\frac{F_1(t)}{F_2(t)}\right)^{\frac{1}{\xi_4}},\xi\equiv\frac{\xi_1}{\xi_2}. \quad (15)$$

From (14) and (15), we solve variables $p_j(t)$ as functions of $F_j(t)$ and $\tilde{y}(t)$. We will find these equations by specifying the parameters. Suppose that these solutions are expressed as follows:

$$p_j(t)=G_j\left(F_1(t),F_2(t),\tilde{y}(t)\right).$$

The profit is now given by:

$$\pi_j(t)=G_j\left(F_1(t),F_2(t),\tilde{y}(t)\right)F_j(t)-r_\delta(t)K_j(t)-w(t)N_j(t). \quad (16)$$

Monopoly j Maximizes the profit with capital and labor as the choice variables. The marginal conditions are

$$\frac{\partial\pi_j}{\partial K_j}=\left(F_j\frac{\partial G_j}{\partial F_j}+G_j\right)\frac{\alpha_j F_j}{K_j}-\left(1+\frac{F_j}{\bar{N}}\frac{\partial G_j}{\partial\tilde{y}}\right)r_\delta=0,$$

$$\frac{\partial\pi_j}{\partial N_j}=\left(F_j\frac{\partial G_j}{\partial F_j}+G_j\right)\frac{\beta_j F_j}{N_j}-\left(1+\frac{F_j}{\bar{N}}\frac{\partial G_j}{\partial\tilde{y}}\right)w=0, \quad (17)$$

in which where we omit time in the expressions. By these equations each monopoly determines the labor and capital inputs as functions of the wage rate, the rate of interest and wealth. The price and output and of monopoly product are then determined separately by (10) and (17). The monopoly's profit is given by (16). How to further analyze behavior of monopoly is referred to some standard microeconomic textbooks (e.g., Mas-Colell, *et.al.*, 1995 [2]).

Demand and supply of final goods

As change in capital stock equal to the output of the final goods sector is minus the depreciation of capital stock and total consumption, we have

$$\dot{K}(t)=F_i(t)-C_i(t)-\delta_k K(t), \quad (18)$$

where $C_i(t)=c_i(t)\bar{N}$.

Labor and capital being fully utilized

The labor market clearing conditions equate labor supply and labor demand. We have:

$$N_i(t)+N_1(t)+N_2(t)=h\bar{N}. \quad (19)$$

For capital markets we have:

$$K_i(t)+K_1(t)+K_2(t)=\bar{k}(t)\bar{N}. \quad (20)$$

We built the model. The model is based on the Solow–Uzawa model, theory of monopoly and Zhang's concept of disposable income and utility function. We now study properties of the model.

3. Properties of the model

The previous section developed a growth model with perfect competition and monopoly. We now provide a computational program to determine the movement of the economic system. We introduce

$$z(t)\equiv\frac{r(t)+\delta_k}{w(t)}.$$

Lemma

The dynamics of the economic system is given by following differential equation:

$$\dot{z}(t)=\bar{\varphi}(z(t)), \quad (21)$$

where function $\bar{\varphi}(z(t))$ is defined in Appendix A-2. All the other variables are explicitly given as functions of $z(t)$ as follows: $\bar{k}(t)$ by (A10) $\rightarrow$ $\rightarrow K(t)=\bar{k}(t)\bar{N} \rightarrow r(t)$ by (A2) $\rightarrow w(t)$ by (A3) $\rightarrow$ $\rightarrow K_1(t)$ and $K_2(t)$ by (A8) $\rightarrow K_i(t)$ by (A9) $\rightarrow N_1(t)$, $N_2(t)$ and $N_i(t)$ by (A1) $\rightarrow F_i(t)$ and $F_j(t)$ by (A6) $\rightarrow$ $\rightarrow p_j(t)$ by (A6) $\rightarrow \pi_j(t)$ by (18) $\rightarrow \hat{y}(t)$ by (4) $\rightarrow c_j(t)$ and $s(t)$ by (7).

We now examine behavior of the economy. It is difficult to give a general solution of the problem. We specify $\xi_0 = 1/2$. In the rest of the paper we are concerned with equilibrium as the genuine dynamic analysis is difficult to be done. In the case of $\xi_0 = 1/2$ by (14) and (15) we have:

$$p_1^2+2a_1p_1-\bar{a}_1=0, \quad (22)$$

where

$$a_1\equiv\lambda_1\xi+\lambda_1\left(\frac{F_2}{F_1}\right)^{0.5},$$

$$\bar{a}_1\equiv\left(\frac{\bar{\xi}_1^2}{1+\lambda_0}\right)\frac{\tilde{y}\bar{N}}{F_1},$$

$$\lambda_1\equiv\frac{\bar{\xi}_2\bar{\xi}_1\lambda_0}{2(1+\lambda_0)}.$$

Solve (22) and (15)

$$p_1(F_1,F_2,z,\bar{k})=-a_1+a_0^{1/2}, a_0\equiv a_1^2+\bar{a}_1,$$

$$p_2(F_1,F_2,z,\bar{k})=\frac{p_1}{\xi}\left(\frac{F_1}{F_2}\right)^{1/2}. \quad (23)$$

With the marginal conditions for capital in (17) and (23) we get:

$$H_j(K_1,K_2,z,\bar{k})\equiv\left(F_j\frac{\partial p_j}{\partial F_j}+p_j\right)\frac{\alpha_jF_j}{K_j}-$$
$$-\left(1-\frac{F_j}{\bar{N}}\frac{\partial p_j}{\partial\tilde{y}}\right)r_\delta=0, \quad (24)$$

in which

$$\frac{\partial p_1}{\partial F_1}=-\frac{\partial a_1}{\partial F_1}+\frac{1}{2a_0^{0.5}}\left(2a_1\frac{\partial a_1}{\partial F_1}+\frac{\partial\bar{a}_1}{\partial F_1}\right),$$
$$\frac{\partial p_1}{\partial\tilde{y}}=\frac{1}{2a_0^{0.5}}\frac{\partial\bar{a}_1}{\partial\tilde{y}},$$

$$\frac{1}{p_2}\frac{\partial p_2}{\partial F_2}=-\frac{2}{F_2}+\frac{1}{p_1}\left(-\frac{\partial a_1}{\partial F_2}+\frac{a_1}{a_0^{0.5}}\frac{\partial a_1}{\partial F_2}\right),$$
$$\frac{1}{p_2}\frac{\partial p_2}{\partial\tilde{y}}=\frac{1}{p_1}\frac{\partial p_1}{\partial\tilde{y}}.$$

The equilibrium condition for (8) means:

$$H_k(K_1,K_2,z,\bar{k})\equiv\frac{\lambda_0\tilde{P}\hat{y}}{\bar{P}+\lambda_0\tilde{P}}-\bar{k}=0. \quad (25)$$

where we use the definition of $\hat{y}$. From (A10) and (28) we have

$$H_i(K_1,K_2,z,\bar{k})\equiv\left(1-\frac{\bar{\beta}_i}{z\bar{\beta}_1}\right)K_1+$$
$$+\left(1-\frac{\bar{\beta}_i}{z\bar{\beta}_2}\right)K_2-\bar{k}\bar{N}+\frac{\bar{\beta}_ih\bar{N}}{z}=0. \quad (26)$$

From (27) and (29) we have three equations to determine three variables, K_1, K_2, and z. To determine the equilibrium values of the economic system we specify the rest parameters as follows:

$$\bar{N}=50, h=4, A_i=1, A_1=1.5, A_2=1.3, \alpha_i=0.33,$$
$$\alpha_1=0.36,\ \alpha_2=0.35,$$

$$\lambda_0=1, \xi_0=0.5, \xi_i=0.6, \xi_1=0.2,$$
$$\xi_2=0.2, \delta_k=0.03. \quad (27)$$

The population is 40 and human capital is 4. Although the specified values of the parameters are not referred to any given economy, we can get insights into economic mechanism of growth by studying effects of different values of these parameters on the national economy. The simulation identifies an equilibrium point. The equilibrium values are as follows:

$$Y = 219,\ K = 211.6,\ F_i = 189.6,\ F_1 = 12.5,\ F_2 = 8.98,\ N_i = 186.6,$$

$$N_1 = 8.19,\ N_2 = 6.79,\ K_i = 195.9,\ K_1 = 8.59,\ K_2 = 7.13,\ \pi_1 = 7.59,\ \pi_2 = 6.58,$$

$$r = 0.29,\ w = 0.69,\ p_1 = 1.27,\ p_2 = 1.5,\ \hat{y} = 8.46,\ \bar{k} = 4.23,\ c_i = 4.63,$$

$$c_1 = 0.25,\ c_2 = 0.18,\ U = 7.49. \quad (28)$$

In (28), the national income is defined as:

$$Y = F_i + p_1 F_1 + p_2 F_2.$$

We see that final goods sector has zero profit due to perfect competition and monopolies have positive profits. We now study how the equilibrium structure is affected when parameters vary.

4. Comparative Static Analysis

The previous section showed growth equilibrium of the national economy with perfect and monopoly product markets. We now examine how the national economy is affected when some exogenous conditions such as preference and technologies are changed. As the Lemma provides a computational procedure to calibrate the model, it is straightforward for us to examine effects of changes in any parameter on the equilibrium values of the economic system. We define a variable $\bar{\Delta}x$ to represent the change rate of the variable x in percentage due to changes in the parameter value.

A monopoly's total factor productivity is enhanced

We first study what happen to the economic system if monopoly 1's total factor productivity is enhanced as follows: $A_1 = 1.5$ to 1.6. The effects on the variables are listed in (29). The output of monopoly product 1 is increaed. Monopoly 1 employs more capital and labor force. Monopoly 2's output is reduced. Monopoly 2 employs less labpr force bur more capital. Monopoly 1 earns more profits, while monopoly 2 earns less profit. The final goods sector produces less. It employs less work force but more capital. The national income and national physical capital are enhanced. The wage rate is increased, while the rate of interest is reduced. The consumer consumes more monopoly product 1, but less final goods and monopoly product 2. The prices of monopoly products are reduced. The utility is enhanced. The household has more wealth and disposable income.

$$\bar{\Delta}Y = 0.34,\ \bar{\Delta}K = 0.32,\ \bar{\Delta}F_i = -0.12,\ \bar{\Delta}F_1 = 13,\ \bar{\Delta}F_2 = -0.14,$$

$$\bar{\Delta}N_i = -0.22,\ \bar{\Delta}N_1 = 5.85,\ \bar{\Delta}N_2 = -0.24,\ \bar{\Delta}K_i = 0.08,\ \bar{\Delta}K_1 = 6.2,\ \bar{\Delta}K_2 = 0.05,$$

$$\bar{\Delta}\pi_1 = 6.5,\ \bar{\Delta}\pi_2 = -0.15,\ \bar{\Delta}r = -0.22,\ \bar{\Delta}w = 0.1,\ \bar{\Delta}p_1 = -6,\ \bar{\Delta}p_2 = -0.01,$$

$$\bar{\Delta}\hat{y} = 0.32,\ \bar{\Delta}\bar{k} = 0.32,\ \bar{\Delta}c_i = -0.16,\ \bar{\Delta}c_1 = 13,\ \bar{\Delta}c_2 = -0.14,\ \bar{\Delta}U = 1.13. \quad (29)$$

The share parameter of a monopoly product is increased

We now examine what happen to the economic system if the share of monopoly product 1 is enhanced as follows: $\xi_1 = 0.2$ to 0.22. The effects on the variables are listed in (30). The output of monopoly product 1 is increaed. Monopoly 1 employs more capital and labor force. Monopoly 2's output is reduced. Monopoly 2 employs less labpr force bur more capital. Monopoly 1 has more profits, while monopoly 2 has less profit. The final goods sector produces less. It employs less work force but more capital. The national income and national physical capital are enhanced. The wage rate is increased, while the rate of interest is reduced. The consumer consumes more monopoly product 1, but less final goods and monopoly product 1. The price of monopoly product 1 is increased, while the price of monopoly product 2 is reduced. The utility is enhanced. The household has more wealth and disposable income. We see that as far as the directions of change are concerned, the rise in the share parameter has similar effects on the economic system as the rise in the total productivity factor, except the effects on the price.

$$\bar{\Delta}Y = 1.1,\ \bar{\Delta}K = 1,\ \bar{\Delta}F_i = -0.37,\ \bar{\Delta}F_1 = 18.5,\ \bar{\Delta}F_2 = -0.4,$$

$$\bar{\Delta}N_i = -0.7,\ \bar{\Delta}N_1 = 18.1,\ \bar{\Delta}N_2 = -0.75,\ \bar{\Delta}K_i = 0.24,\ \bar{\Delta}K_1 = 19.2,\ \bar{\Delta}K_2 = 0.17,$$

$$\bar{\Delta}\pi_1 = 20.5,\ \bar{\Delta}\pi_2 = -0.47,\ \bar{\Delta}r = -0.7,\ \bar{\Delta}w = 0.3,$$
$$\bar{\Delta}p_1 = 0.8,\ \bar{\Delta}p_2 = -0.03,$$

$$\bar{\Delta}y = 1,\ \bar{\Delta}\bar{k} = 1,\ \bar{\Delta}c_i = -0.5,\ \bar{\Delta}c_1 = 18.5,$$
$$\bar{\Delta}c_2 = -0.43,\ \bar{\Delta}U = 3.6. \quad (30)$$

The share parameter of final goods is increased

We now examine what happen to the economic system if the share of final goods is enhanced as follows: ξ_i = 0.6 to 0.61. The effects on the variables are listed in (31). The output of the final goods sector is increased. The final goods sector employs more capital and labor force. The output of two monopoly products are reduced. The two monopolies employ less capital and labor force. They earn less profits. The national income and national physical capital are decreated. The wage rate is reduced, while the rate of interest is enhanced. The consumer consumes less monopoly products, but more final goods. The prices of monopoly products are decreased. The utility is enhanced. The household has less wealth and disposable income.

$$\bar{\Delta}Y = -0.31,\ \bar{\Delta}K = -0.29,\ \bar{\Delta}F_i = 0.11,$$
$$\bar{\Delta}F_1 = -2.88,\ \bar{\Delta}F_2 = -2.9,$$

$$\bar{\Delta}N_i = 0.2,\ \bar{\Delta}N_1 = -2.8,\ \bar{\Delta}N_2 = -2.8,\ \bar{\Delta}K_i = -0.1,$$
$$\bar{\Delta}K_1 = -3.1,\ \bar{\Delta}K_2 = -3.1,$$

$$\bar{\Delta}\pi_1 = -3.1,\ \bar{\Delta}\pi_2 = -3.1,\ \bar{\Delta}r = 0.2,\ \bar{\Delta}w = -0.1,$$
$$\bar{\Delta}p_1 = -0.12,\ \bar{\Delta}p_2 = -0.1,$$

$$\bar{\Delta}\hat{y} = -0.29,\ \bar{\Delta}\bar{k} = -0.29,\ \bar{\Delta}c_i = 0.14,$$
$$\bar{\Delta}c_1 = -2.9,\ \bar{\Delta}c_2 = -2.9,\ \bar{\Delta}U = 2.3. \quad (31)$$

The propensity to save is enhanced

We first study what happen to the economic system if the propensity to save is increased as follows: λ_0 = 1 to 1.05. The effects on the variables are listed in (32). The household has more wealth and disposable income. The national income and physical capital are increased. The final goods sector and two monopolies produce more and employ more capital. The final sector's labor force is not changed. Monopoly 1 employs more labor force but monopoly 2 employs less labor force. The wage rate is increased, while the rate of interest is reduced. The prices of monopoly products are redeuced. The household consumes more and has more wealth and disposable income.

$$\bar{\Delta}Y = 2.4,\ \bar{\Delta}K = 7.3,\ \bar{\Delta}F_i = 2.4,$$
$$\bar{\Delta}F_1 = 2.6,\ \bar{\Delta}F_2 = 2.5,$$

$$\bar{\Delta}N_i = 0,\ \bar{\Delta}N_1 = 0.03,\ \bar{\Delta}N_2 = -0.03,\ \bar{\Delta}K_i = 7.2,$$
$$\bar{\Delta}K_1 = 7.4,\ \bar{\Delta}K_2 = 7.3,$$

$$\bar{\Delta}\pi_1 = 2.4,\ \bar{\Delta}\pi_2 = 2.3,\ \bar{\Delta}r = -5.1,\ \bar{\Delta}w = 2.4,$$
$$\bar{\Delta}p_1 = -0.21,\ \bar{\Delta}p_2 = -0.14,$$

$$\bar{\Delta}\hat{y} = 4.8,\ \bar{\Delta}\bar{k} = 7.3,\ \bar{\Delta}c_i = 2.2,\ \bar{\Delta}c_1 = 2.6,$$
$$\bar{\Delta}c_2 = 2.5,\ \bar{\Delta}U = 18.3. \quad (32)$$

The final goods sector's total factor productivity is enhanced

We now examine what happen to the economic system if the final goods sector's total factor productivity is enhanced as follows: A_i = 1 to 1.1. The effects on the variables are listed in (33). The output of the final goods sector is increased. The final goods sector employs more capital and labor force. The output of two monopoly products are reduced. The two monopolies employ less labor force but more capital. They earn more profits. The national income and national physical capital are decreated. The wage rate and the rate of interest are enhanced. The consumer consumes less monopoly products, but more final goods. The prices of monopoly products are increased. The utility is enhanced. The household has more wealth and disposable income.

$$\bar{\Delta}Y = 14.3,\ \bar{\Delta}K = 14.4,\ \bar{\Delta}F_i = 15.6,$$
$$\bar{\Delta}F_1 = -2.9,\ \bar{\Delta}F_2 = -3.3,$$

$$\bar{\Delta}N_i = 0.55,\ \bar{\Delta}N_1 = -7.5,\ \bar{\Delta}N_2 = -7.7,\ \bar{\Delta}K_i = 15.1,$$
$$\bar{\Delta}K_1 = 5.8,\ \bar{\Delta}K_2 = 5.6,$$

$$\bar{\Delta}\pi_1 = 5.5,\ \bar{\Delta}\pi_2 = 0.55,\ \bar{\Delta}r = 15,$$
$$\bar{\Delta}w = 9.2,\ \bar{\Delta}p_1 = 9.4,\ \bar{\Delta}p_2 = 14.4,$$

$$\bar{\Delta}\hat{y} = 14.4,\ \bar{\Delta}\bar{k} = 14.4,\ \bar{\Delta}c_i = 15.7,$$
$$\bar{\Delta}c_1 = -2.9,\ \bar{\Delta}c_2 = -3.3,\ \bar{\Delta}U = 29.3. \quad (33)$$

A monopoly's output elasticity of labor is enhanced

We now analyze what happen to the economic system if monopoly 1's product elasticities of capital and labor are changed respectively as follows: α_1 = 0.37 to 0.36 and β_1 = 0.63 to 0.64. The effects on the variables are listed in (34).

The output of monopoly product 1 is increaed. Monopoly 1 employs more capital and labor force. Monopoly 2's output is reduced. Monopoly 2 employs less labpr force and capital. Monopoly 1 earns more profits, while monopoly 2 earns less profit. The final goods sector produces less. It employs less work force and capital. The national income and national physical capital are enhanced. The wage rate is decreased, while the rate of interest is increased. The consumer consumes more monopoly product 1, but less final goods and monopoly product 2. The prices of monopoly products are reduced. The utility is enhanced. The household has more wealth and disposable income.

$$\begin{aligned}
&\bar{\Delta}Y = 0.16,\ \bar{\Delta}K = 0.07,\ \bar{\Delta}F_i = -0.2,\\
&\bar{\Delta}F_1 = 9.7,\ \bar{\Delta}F_2 = -0.3,\\
&\\
&\bar{\Delta}N_i = -0.2,\ \bar{\Delta}N_1 = 7.3,\ \bar{\Delta}N_2 = -0.3,\ \bar{\Delta}K_i = -0.23,\\
&\bar{\Delta}K_1 = 7.2,\ \bar{\Delta}K_2 = -0.34,\\
&\\
&\bar{\Delta}\pi_1 = 1.6,\ \bar{\Delta}\pi_2 = -0.3,\ \bar{\Delta}r = 0.05,\ \bar{\Delta}w = -0.02,\\
&\bar{\Delta}p_1 = -4.7,\ \bar{\Delta}p_2 = -0.002,\\
&\\
&\bar{\Delta}\hat{y} = 0.07,\ \bar{\Delta}\bar{k} = 0.07,\ \bar{\Delta}c_i = -0.3,\ \bar{\Delta}c_1 = 9.7,\\
&\bar{\Delta}c_2 = -0.3,\ \bar{\Delta}U = 0.5.
\end{aligned} \tag{34}$$

We also conducted comparative analyses for the population and human capital. The population change causes proportional changes in macroeconomic real variables, almost no effect on prices and microeconomic real variables. A rise in human capital causes rises in macroeconomic and microeconomic real variables, no change in the prices, and rises in the monopolies' profits.

5. Conclusion

The purpose of this study is to introduce monopolies to neoclassical growth theory with Zhang's concept of disposable income and utility function. This unique contribution makes neoclassical economic growth theory more realistic in modelling the complexity of market structures. It integrates neoclassical growth theory with the basic economic mechanisms in monopoly theory. We introduce perfect competition, monopolistic competition, and monopoly to traditional growth The model is based on a few well-established economic theories in the literature of economics. We framed the model on basis of the Solow–Uzawa two-sector growth model. The modelling of monopoly is based on monopoly theory well developed in the literature of economics. We modelled behavior of the household with Zhang's concept of disposable income and utility function. This research is to integrate these theories in a comprehensive framework. It endogenously determines profits of monopolies which are equally distributed among the homogeneous population. We built the model and then identified the existence of an equilibrium point by simulation. We conducted comparative static analyses in some parameters. As an initial integration of different theories within a compact framework and each theory has its own complicated literature, it is not difficult to conceptually and analytically extend and generalize our model. It is straightforward to generalize the model by introducing more goods in competitive markets and more monopolies. We can also introduce monopolistic competition into the analytical framework developed in this study (e.g., Dixit and Stiglitz, 1977 [18]; Romer, 1990 [20]; Wang, 2012 [4]; and Zhang, 2018 [23]).

Appendix

A1: Solving the Consumer Problem

We now maximize utility (6) subject to budget constraint (5). We form the Lagrangian function as follows:

$$\begin{aligned}
L = &\left(\xi_1 c_1^{\xi_0} + \xi_2 c_2^{\xi_0} + \xi_i c_i^{\xi_0}\right)^{1/\xi_0} s^{\lambda_0} + \\
&+ b\left(\hat{y} - p_1 c_1 - p_2 c_2 - c_i - s\right).
\end{aligned} \tag{A1.1}$$

Maximizing L, we get

$$\frac{\partial L}{\partial c_j} = \frac{\xi_j c_j^{\xi_0 - 1}}{\xi_1 c_1^{\xi_0} + \xi_2 c_2^{\xi_0} + \xi_i c_i^{\xi_0}} - \frac{p_j b}{U} = 0, j = 1, 2, \tag{A1.2}$$

$$\frac{\partial L}{\partial c_i} = \frac{\xi_i c_i^{\xi_0 - 1}}{\xi_1 c_1^{\xi_0} + \xi_2 c_2^{\xi_0} + \xi_i c_i^{\xi_0}} - \frac{b}{U} = 0, \tag{A1.3}$$

$$\frac{\partial L}{\partial s} = \frac{\lambda_0}{s} - \frac{b}{U} = 0, \tag{A1.4}$$

$$\frac{\partial L}{\partial b} = \hat{y} - p_1 c_1 - p_2 c_2 - c_i - s = 0. \tag{A1.5}$$

From (A1.2) and (A1.3) we have

$$\frac{c_j}{c_i} = P_j \equiv \bar{\xi}_j^{-\xi_4} p_j^{\xi_4}, \tag{A1.6}$$

where $\bar{\xi}_j \equiv \xi_j / \xi_i$ and $\xi_4 \equiv 1/(\xi_0 - 1)$.

Insert (A1.6) in (A1.3)

$$\frac{1}{\bar{\xi}_1 P_1^{\xi_0} + \bar{\xi}_2 P_2^{\xi_0} + 1} - \frac{c_i b}{U} = 0. \quad \text{(A1.7)}$$

From (A1.4)-(A1.7) we solve:

$$\frac{U}{b} = \frac{\tilde{P}\hat{y}}{\bar{P} + \lambda_0 \tilde{P}}, \quad \text{(A1.8)}$$

where

$$\bar{P} \equiv p_1 P_1 + p_2 P_2 + 1, \tilde{P} \equiv \bar{\xi}_1 P_1^{\xi_0} + \bar{\xi}_2 P_2^{\xi_0} + 1.$$

From (A1.8) and (A1.4), we have:

$$s = \frac{\lambda_0 \tilde{P}\hat{y}}{\bar{P} + \lambda_0 \tilde{P}}. \quad \text{(A1.9)}$$

$$c_i = \frac{\hat{y}}{\bar{P} + \lambda_0 \tilde{P}}, \quad \text{(A1.10)}$$

$$c_j = \frac{P_j \hat{y}}{\bar{P} + \lambda_0 \tilde{P}}. \quad \text{(A1.11)}$$

A2: Confirming the Lemma

From (3) and (17) we get

$$z \equiv \frac{r + \delta_k}{w} = \frac{\bar{\beta}_i N_i}{K_i} = \frac{\bar{\beta}_j N_j}{K_j}, \quad \text{(A2.1)}$$

where $\bar{\beta}_x \equiv \alpha_x / \beta_x$. By (2) we have

$$r(z) = \alpha_i A_i \left(\frac{z}{\bar{\beta}_i} \right)^{\beta_i} - \delta_k. \quad \text{(A2.2)}$$

From (A2.1), we have:

$$w = \frac{r + \delta_k}{z}. \quad \text{(A2.3)}$$

By (2), (10), and (A1), we have:

$$F_x = A_x N_x \left(\frac{\bar{\beta}_x}{z} \right)^{\alpha_x}. \quad \text{(A2.4)}$$

From (A2.1) and (A2.4) we have:

$$K_x = \frac{\bar{\beta}_x N_x}{z}. \quad \text{(A2.5)}$$

By (14) we have

$$F_j - \frac{P_j \left(\tilde{y} \bar{N} + p_1 F_1 + p_2 F_2 \right)}{\bar{P} + \lambda_0 \tilde{P}} = 0. \quad \text{(A2.6)}$$

We assume that (A2.6) has a solution $p_j = G_j(F_j, \tilde{y})$. By (19), we have:

$$\left(F_j \frac{\partial G_j}{\partial F_j} + G_j \right) \frac{\alpha_j F_j}{K_j} = \left(1 + \frac{F_j}{\bar{N}} \frac{\partial G_j}{\partial \tilde{y}} \right) (r + \delta_k). \quad \text{(A2.7)}$$

From (A2.7) and (A2.2) and (A2.3) which express r and w as functions of z, we solve the two variables as follows:

$$K_j = \bar{G}_j(z, \bar{k}). \quad \text{(A2.8)}$$

Insert (A2.1) in (19)

$$K_i + \frac{\bar{\beta}_i K_1}{\bar{\beta}_1} + \frac{\bar{\beta}_i K_2}{\bar{\beta}_2} = \frac{\bar{\beta}_i h \bar{N}}{z}. \quad \text{(A2.9)}$$

Insert (A2.9) in (20)

$$H(z, \bar{k}) \equiv \left(1 - \frac{\bar{\beta}_i}{\bar{\beta}_1} \right) \bar{G}_1 + \left(1 - \frac{\bar{\beta}_i}{\bar{\beta}_2} \right) \bar{G}_2 - \bar{k}\bar{N} + \frac{\bar{\beta}_i h \bar{N}}{z} = 0. \quad \text{(A2.10)}$$

Assume that $H(z, \bar{k}) = 0$ has a solution, given by $\bar{k} = \varphi(z)$. It is straightforward to confirm that all the variables can be expressed as functions of z by the following procedure: $\bar{k}$ by (A10) $\rightarrow K = \bar{k}\bar{N}$ $\rightarrow r$ by (A2) $\rightarrow w$ by (A3) $\rightarrow K_1$ and K_2 by (A8) $\rightarrow K_i$ by (A9) $\rightarrow N_1$, N_2 and N_i by (A1) $\rightarrow F_i$ and F_j by (A6) $\rightarrow p_j$ by (A6) $\rightarrow \pi_j$ by (18) $\rightarrow \hat{y}$ by (4) $\rightarrow c_i$, c_j and s by (7). From this procedure and (8) we have:

$$\dot{\bar{k}} = f(z) \equiv s(z) - \varphi(z). \quad \text{(A10)}$$

Derive $\bar{k} = \varphi(z)$ in time:

$$\dot{\bar{k}} = \frac{d\varphi}{dz} \dot{z}. \quad \text{(A11)}$$

From (A11) and (A10), we have:

$$\dot{z} = \left(\frac{d\varphi}{dz} \right)^{-1} f. \quad \text{(A12)}$$

In summary, we proved the Lemma.

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